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Branes in AdS and pp-wave spacetimes

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ABSTRACT

We find half supersymmetric AdS-embeddings in $AdS_5 \times S^5$ corresponding to all quarter BPS orthogonal intersections of D3-branes with Dp-branes. A particular case is the Karch-Randall embedding $AdS_4 \times S^2$. We explicitly prove that these embeddings are supersymmetric by showing that the kappa symmetry projections are compatible with half of the target space Killing spinors and argue that all these cases lead to AdS/dCFT dualities involving a CFT with a defect. We also find an asymptotically $AdS_4 \times S^2$ embedding that corresponds to a holographic RG-flow on the defect. We then consider the pp-wave limit of the supersymmetric AdS-embeddings and show how it leads to half supersymmetric D-brane embeddings in the pp-wave background. We systematically analyze D-brane embeddings in the pp-wave background along with their supersymmetry. We construct all supersymmetric D-branes wrapped along the light-cone using operators in the dual gauge theory: the open string states are constructed using defect fields. We also find supersymmetric D1 (monopoles) and D3 (giant gravitons) branes that wrap only one of the light-cone directions. These correspond to non-perturbative states in the dual gauge theory.

Contents

1 Introduction and summary of results

Two of the most interesting recent developments are the extension of the AdS/CFT duality to conformal field theories with a defect (AdS/dCFT duality) [1, 2, 3, 4, 5, 6, 7, 8], and the study of the pp-wave limit of AdS backgrounds [9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28]. In the former case one adds additional structure on both sides of the duality: a D-brane in the bulk and a defect in the boundary theory. The theory on the defect captures holographically the physics of the D-brane in the bulk, and the interactions between the bulk and the D-brane modes are encoded in the couplings between the boundary and the defect fields.

The pp-wave limit of an AdS background is a special case of a Penrose limit of a gravitational background [29, 30, 9, 10]. The particular interest in this Penrose limit stems from the fact that one can combine the limit with the AdS/CFT duality to obtain a relation between string theory on a pp-wave background and a specific limit of the dual conformal field theory [11]. It turns out that string theory on the pp-wave background is exactly solvable [31, 32, 33] and this raises the possibility of understanding quantitatively holography for a background which is very close to flat space.

In this paper we give support to and propose a whole host of new AdS/dCFT dualities by studying D-brane embeddings in $AdS_5 \times S^5$ and analyzing their supersymmetry. We show that the supersymmetric AdS-embeddings that we find lead to supersymmetric D-brane embeddings on pp-wave backgrounds. Utilizing the AdS/dCFT duality, we construct the light-cone D-brane states using gauge theory operators for all D-branes.

In the AdS/dCFT duality proposed by Karch and Randall [3], one considers a D5 brane wrapping an $AdS_4 \times S^2$ submanifold of $AdS_5 \times S^5$. This configuration may be considered as the near-horizon limit of a certain D3-D5 system, and the AdS/CFT duality is considered to act twice: both in the bulk and on the worldvolume. In the limit discussed in [4], the bulk description is in terms of supergravity coupled to a probe D5-brane. The dual theory is $\mathcal{N} = 4$ SYM theory coupled to a three dimensional defect. The defect theory may be associated with the boundary of AdS_4, and as such it should be a conformal field theory.

An important issue is whether the $AdS_4 \times S^2$ embedding is stable and supersymmetric. In the probe computation of [3] it was found that the D-brane configuration is sitting in the maximum of the potential and that there is a tachyonic mode. The mass of the tachyonic mode was shown to be above the Breitenlohner-Freedman bound [34], signaling stability. Further evidence for stability and supersymmetry of the configuration was provided in [4] where it was shown that one can fit the KK bosonic fields of the S^2-reduction of the worldvolume theory in appropriate supermultiplets. We prove in the paper that the configuration is supersymmetric, thus dispelling any doubts about the stability of the system. In particular, we show that the $AdS_4 \times S^2$ configuration with or without flux on the S^2 preserves 16 supercharges by showing that the worldvolume kappa symmetry projection is compatible with 16 of the 32 target space Killing spinors.

In the AdS/CFT correspondence one may study holographic RG-flows by considering solutions that preserve d-dimensional Poincare invariance and are asymptotically AdS_{d+1}. The asymptotic behavior of the bulk field shows whether the solution describes an operator deformation or a new (non-conformal) vacuum of the CFT. One expects a similar story in the AdS/dCFT duality. In particular, one may deform the $\mathcal{N}=4$ SYM and/or the defect CFT. In the former case, one should study D5 embeddings on the asymptotically AdS solution describing the RG-flow. The embeddings are expected to only be asymptotically AdS, signaling an induced RG-flow on the defect theory. This is an interesting subject which, however, we will not pursue further in this paper.

The second possibility is to only deform the defect CFT. Within the approximations used in this paper, the D5-brane theory does not backreact on the bulk. This means that the boundary theory remains conformal, but the defect theory runs. This situation should be described by an asymptotically $AdS_4 \times S^2$ embedding in $AdS_5 \times S^5$. We indeed find such an embedding, where only the AdS_4 part of the solution is deformed. This means that the defect QFT still has the same R-symmetry as the defect CFT. Furthermore, this embedding completely breaks supersymmetry. Using the operator-field dictionary developed in [4], and the asymptotic form of the worldvolume fields we show that the RG-flow corresponds to a vev deformation of the defect theory. The operator that gets the vev is a specific scalar component of a $3d$ superfield, so its vev breaks supersymmetry. Furthermore, it is an R-singlet in accordance with the fact that the S^2 part of the solution is undeformed. An interesting feature of this RG-flow is that the theory develops a mass gap in the infrared. We propose that the vev corresponds to giving masses to the 3-5 strings.

We also find solutions where the D5-brane wraps an S^2 with vanishing volume but non-zero flux. Some of these solutions are supersymmetric and some not but in all cases the solution can be re-interpreted as a D3-brane. The non-supersymmetric cases correspond to either anti-D3 branes or D3-branes misaligned with the background. The supersymmetric case correspond to D3 branes aligned with the D3-branes creating the background. Next we point out that $AdS_{m+1} \times S^{n+1}$ embeddings with general (m,n) in this background satisfy the field equations. They are, however, only supersymmetric when $|m-n|=2$; this follows from the well-known intersection rules for intersecting D-brane systems, as we will discuss further below.

We next consider the pp-wave Penrose limit. In the case of $AdS_5 \times S^5$, the corresponding field theory limit is the large N and J limit, where J is R-charge associated with a specific $SO(2)$ subgroup of the R-symmetry group [11]. In this limit, the authors of [11] (BMN) identified certain field theory operators with closed string states in the pp-wave background. In the duality of [3], the bulk theory involves the degrees of freedom of a single D5 brane, and these are encoded holographically in the defect CFT. This suggests that the corresponding pp-wave limit will capture the degrees of freedom of the D5 brane as well, and that the field theory operators that correspond to open string states on the pp-wave background are defect operators. This observation was also made independently in a nice paper [28] that appeared while this work was finalized. We shall show that all D-branes can be understood in this fashion.

In the pp-wave spacetime four of the transverse coordinates, y^a, originate from AdS_5 and the other four, z^a, from S^5. We denote this splitting of transverse coordinates as $(4,4)$, or more explicitly as $(y^a; z^a)$. We find that the Penrose limit of the $AdS_4 \times S^2$-embeddings in $AdS_5 \times S^5$ are D5-brane embeddings in the pp-wave background which are longitudinal to the light-cone (provided that the original brane wraps the boosted circle). The induced geometry is that of a pp-wave localized in certain coordinates. In particular, the $AdS_4 \times S^2$ embedding with zero flux over the S^2 yields a pp-wave localized at the origin of $(1,3)$, i.e. $(0;0,0,0)$. Turning on a flux q on the S^2 has the effect of shifting the position of the pp-wave in the AdS-direction by an amount equal to the flux, i.e. the D5 brane is now located at $(\pm q;0,0,0)$.

By a direct computation we show that the solution with or without flux preserves half of the supersymmetry, in agreement with the fact that the original brane preserved 16 supercharges. (The Penrose limit always at least preserves supersymmetry.) One can also have D5-brane embeddings located at arbitrary constant positions in the $(1,3)$ directions. These branes, however, preserve only one quarter of the supersymmetry. This is a generic phenomenon: brane embeddings with constant transverse scalars generically preserve 1/4 of supersymmetry. In some cases, however, the supersymmetry is enhanced by a factor of two when the brane is located at special points which indicates that the D-brane configuration originates from a 1/2 supersymmetric AdS-embedding in $AdS_5 \times S^5$. Note the non-supersymmetric $AdS_{m+1} \times S^{n+1}$ embeddings (with $|m-n| \neq 2$) mentioned above lead to 1/4 supersymmetric embeddings in the Penrose limit; there is a supersymmetry enhancement, rather akin to what can happen for non-maximally supersymmetric backgrounds [12, 13, 14].

Motivated by these results we then investigate systematically D-brane embeddings in the pp-wave background and their supersymmetry. We restrict ourselves (mostly) to embeddings with constant transverse scalars and zero flux. In some sense these are the "elementary" embeddings and more complicated ones may be considered as "superpositions" or excitations of the elementary embeddings. The branes can be divided into longitudinal branes where the light-cone coordinates are on the worldvolume, instantonic branes, where the lightcone coordinates are in the transverse coordinates and branes with one lightcone coordinate along the worldvolume and the other transverse to it. In light-cone open string theory, only the first branes are visible, and in light-cone closed string theory one can only construct boundary states for the instantonic ones. Although the last branes are not visible in the light-cone gauge, they should be present in covariant gauges. Except for a few exceptional cases (to be discussed in the main text), we find solutions for all possible splittings of (constant) transverse and longitudinal coordinates.

All longitudinal and instantonic branes preserve 1/4 of supersymmetry, unless the coordinate splitting is special and the branes are sitting at the origin of the transverse space. In this case we have a solution that preserves 16 supercharges. As discussed in the previous paragraph, these cases indicate that the solution originates from the 1/2 supersymmetric AdS-embedding on $AdS_5 \times S^5$. The latter embeddings, in turn, are expected to originate from the near-horizon limit of supersymmetric intersections of D3 branes with other Dp-branes. Indeed, inspection of our results confirms this picture and can be summarized in the following Table 1.

In the first column of Table 1 we indicate the D-brane embedding. The second column lists the supersymmetric intersection of the D-brane under consideration with D3-branes. The top part of the table contains supersymmetric intersections where the number of Neumann-Dirichlet (ND) boundary conditions is equal to four. These intersections are also sometimes called standard intersections. The bottom part of the table contains supersymmetric intersections with number of ND boundary conditions equal to eight (non-standard intersections). The notation $(r|Dp \perp D3)$ means that a Dp brane intersects orthogonally a $D3$ brane over an r-brane. The third column indicates the expected half supersymmetric D-brane embeddings in $AdS_5 \times S^5$. These embeddings follow from the second column by considering the near-horizon limit of the D3-branes and taking the Dp brane to extend in the radial direction of the D3 brane and have the remaining $(p-r-1)$ worldvolume direc-

Brane	$ND = 4$ intersections	Embedding	Longitudinal
D1	$(0\|D1 \perp D3)$	AdS_2	-
D3	$(1\|D3 \perp D3)$	$AdS_3 \times S^1$	$(+,-,2,0)$
D5	$(2\|D5 \perp D3)$	$AdS_4 \times S^2$	$(+,-,3,1)$
D7	$(3\|D7 \perp D3)$	$AdS_5 \times S^3$	$(+,-,4,2)$
	$ND = 8$ intersections		
D5	$(0\|D5 \perp D3)$	$AdS_2 \times S^4$	$(+,-,1,3)$
D7	$(1\|D7 \perp D3)$	$AdS_3 \times S^5$	$(+,-,2,4)$

Table 1: *Half supersymmetric branes in pp wave backgrounds and their relation to brane intersections and half supersymmetric branes in* $AdS_5 \times S^5$.

tions wrapped on an S^{p-r-1} sphere in S^5. The fourth column lists the half supersymmetric longitudinal Dp-branes that we find in this paper, for which we use the notation $(+,-,m,n)$ to denote the worldvolume directions. Note that the AdS_2 embedding necessarily leads to a pp-wave D-brane wrapping the x^--light-cone direction, and thus decouples in the pp-wave limit. Instantonic branes can be obtained from the longitudinal ones by formal T-duality along the light-cone directions. The results so obtained agree with the results obtained in [19] using the boundary state approach.

The pp-wave background is symmetric under interchange of y^a by z^a. This implies that the supersymmetric branes should come in pairs, (p,q) and (q,p) and we indeed find that the results in Table 1 have this property, except for one entry. We seem to be missing the the $D3$-$(+,-,0,2)$ brane. Inspection of the table suggests that such a brane would have come from a "AdS_1"$\times S^3$ embedding, where "AdS_1" stands for the time-like direction R. There is one $ND = 8$ intersection that we have not mentioned so far because of its unusual reality properties: $(-1|D3 \perp D3)$. This intersection leads to an instantonic $D3$-$(1,3)$-brane in the pp-wave limit. We leave as an open problem to find the origin of the $D3$-$(+,-,0,2)$ brane on the AdS-side.

Branes which lie along only one direction of the lightcone must be rotating around the boosted circle. Branes which lie along the x^- direction, which would be called $(-,m,n)$ branes in the above notation, are shown to have degenerate induced metrics and are not admissible embeddings; they have infinite energy as measured by the lightcone Hamiltonian. If we take the Penrose limit of any $AdS_{m+1} \times S^{n+1}$ brane boosting along a circle orthogonal to the brane, under the infinite boost the brane will be mapped to a $(-, m, n)$ brane and hence disappears from the physical spectrum.

Branes which lie along the x^+ direction are rotating at the speed of light in the direction of the boost. We find a number of such embeddings, and show explicitly that several of them are supersymmetric. In particular, we find 1/4 supersymmetric $D1$-branes, $(+, 1, 0)$ and $(+, 0, 1)$, and the corresponding rotating D-strings in $AdS_5 \times S^5$. We suggest that the dual gauge theory interpretation of these branes is as $SU(N)$ magnetic monopoles[1]. We also find the Penrose limit of the BPS giant graviton branes [35, 36, 37].

Having obtained the supersymmetric AdS-embeddings given in Table 1 one may generalize the arguments of [3, 4] to obtain a new set of AdS/dCFT dualities. In each of the cases listed one may consider the limit described in [4] to obtain a duality between the bulk $AdS_5 \times S^5$ with a single Dp-brane and $d = 4$ $\mathcal{N} = 4$ SYM theory interacting with a defect theory. The defect theory is that of the low energy modes of the 3-p and p-3 strings; operators on the defect are dual to fields on the Dp-brane.

We have argued above that the supersymmetric AdS-embeddings are mapped to D-brane embeddings in the pp-wave background. The duality described in the previous paragraph suggests that the light-cone D-brane states can be constructed by defect field operators. We indeed find this to be the case. We propose that the light-cone open string ground state corresponds to an operator that is the trace of a large number of Z's (as in the closed string sector of [11]) together with a bilinear of the massless modes of the 3-p and p-3 strings. In particular, the massless modes of the 3-p and p-3 string in the $ND = 4$ cases of Table 1 are four hypermultiplets in the fundamental and anti-fundamental of $SU(N)$, and the ground state contains the unique $J = 1$ gauge invariant combination. On the other hand, the massless spectrum of the 3-p and p-3 open strings in the $ND = 8$ configurations consists of two fermions, one in the fundamental and another in the anti-fundamental of $SU(N)$.

The proposal for the light-cone ground states in the two cases is given in (9.1) and (9.3), respectively. In both cases we find that the light-cone energy agrees with the result obtained by a direct computation in the open string theory on the pp-wave background. As already mentioned, the longitudinal branes always come in pairs, $(+, -, p, q)$ and $(+, -, q, p)$. The construction of the former brane goes through an ND=4 system, and the construction of the latter through an ND=8 system. Nevertheless, the symmetry of the background implies that the two should be equivalent. In particular, the ground state energy should be the same and we indeed find both (9.1) and (9.3) have the same light-cone energy! Furthermore,

[1] We thank Erik Verlinde for a discussion about this point.

following [11] and the recent papers [26, 28], we construct the oscillators by inserting $D_i Z$ and ϕ_i operators in closed and open string ground states and by adding phase (closed strings), cosine (open strings with Neumann boundary conditions) and sine (open strings with Dirichlet boundary conditions) factors.

The D5 and D7 branes appear twice in Table 1, and thus in the decoupling limit one obtains two distinct supersymmetric D5/7-brane embeddings into $AdS_5 \times S^5$, with the corresponding dual theories containing different codimension defects. However, as we said, in the pp-wave limit the symmetry of the background under the interchange of the y^a and z^a directions implies that the $(+, -, m, n)$ branes are indistinguishable from the $(+, -, n, m)$ branes. This means the surviving sectors of the two dCFTs dual to the D-branes should be equivalent, even though they contain defects of different dimensions. We leave the study of this rather novel "duality" for future work.

The organisation of the paper is as follows. In §2 we derive the D-brane field equations in full generality, for use in later sections. In §3 we find D5-brane embeddings into the $AdS_5 \times S^5$ background. In §4 we use the kappa symmetry projector to determine the supersymmetry preserved by these D-brane embeddings. In §5 we discuss the holographic interpretation of asymptotically $AdS_4 \times S^2$ embeddings in terms of an RG flow of the defect theory. In §6 we consider the Penrose limits of our brane embeddings, and explicitly verify their supersymmetry in the pp-wave background. In §7 we discuss more generally brane embeddings into $AdS_5 \times S^5$ and their supersymmetry. In §8 we derive brane embeddings in the pp-wave background, considering branes with two, one and zero directions along the light cone. In §9 we present the new AdS/dCFT dualities, and we construct the D-brane states from gauge theory operators. Finally, in appendix A we list our conventions and in appendix B we derive and discuss an alternative form of the kappa symmetry projection used in §4.

2 D-brane field equations

In this section we derive the D-brane field equations in full generality. It is common practice in the literature to substitute an ansatz for a D-brane embedding in the D-brane action and then derive field equations by varying the functions appearing in the ansatz. A given ansatz, however, may not be consistent, i.e. the (components of the) fields that are set to zero may be sourced by non-zero terms in the actual field equations. As the D-brane equations are non-linear, such potential problems are certainly an issue. To avoid such pitfalls, we derive the field equations in all generality and for all Dp-branes in this section. The result, given in (2.21), is rather compact and can be effectively used in actual computations. That is, it is straightforward to obtain the equations satisfied by the functions appearing in a given ansatz by just substituting the ansatz in (2.21). We carry out such computations in the subsequent sections.

The worldvolume action for a single Dp-brane is given by

$$
\begin{aligned}
I_p &= I_{DBI} + I_{WZ} \\
I_{DBI} &= = -T_p \int_M d^{p+1}\xi e^{-\Phi} \sqrt{-\det\left(g_{ij} + \mathcal{F}_{ij}\right)}, \qquad I_{WZ} = T_p \int_M e^{\mathcal{F}} \wedge C,
\end{aligned} \tag{2.1}
$$

with T_p the Dp-brane tension, which henceforth we set to one. Here ξ^i are the coordinates of the $(p+1)$-dimensional worldvolume M which is mapped by worldvolume fields X^m into the target space which has (string frame) metric g_{mn}. This embedding induces a worldvolume metric $g_{ij} = g_{mn}\partial_i X^m \partial_j X^n$. The worldvolume also carries an intrinsic abelian gauge field A with field strength F. $\mathcal{F} = F - B$ is the gauge invariant two-form with $B_{ij} = \partial_i X^m \partial_j X^n B_{mn}$ the pullback of the target space NS-NS 2-form. Note that we set $2\pi\alpha' = 1$ in all that follows. The RR n-form gauge potentials (pulled back to the worldvolume) are collected in

$$
C = \bigoplus_n C_{(n)}, \tag{2.2}
$$

and the integration over M automatically selects the proper forms in this sum. To simplify the notation we will denote target space tensors and their pullbacks by the same letter. One can distinguish between the two by their indices: $m, n, p, \ldots$ denote target space indices, and i, j, k pullbacks. For instance,

$$
A_{ijmn} = \partial_i X^p \partial_j X^q A_{pqmn} \tag{2.3}
$$

where A_{pqmn} denotes some target space tensor.

Now let us derive the equations of motion which follow from (2.1). It is convenient to treat the variation of the DBI and WZ terms separately and to introduce the notation

$$
M_{ij} = (\partial_i X^m \partial_j X^n g_{mn} - \partial_i X^m \partial_j X^n B_{mn} + F_{ij}). \tag{2.4}
$$

Let us define the inverse of M_{ij} such that

$$
M^{ij} M_{jk} = \delta^i{}_k. \tag{2.5}
$$

Then variation of the DBI term gives

$$
\delta I_{DBI} \;=\; -\int d^{p+1}\xi e^{-\Phi}\sqrt{-M}\left(-\Phi_{,m}\delta X^m + \tfrac{1}{2} M^{ji}\delta M_{ij}\right); \tag{2.6}
$$

$$
\begin{aligned}
= & -\int d^{p+1}\xi e^{-\Phi}\sqrt{-M}\left(G^{ij}(\partial_i\delta X^m)(\partial_j X^n)g_{mn}+\theta^{ij}(\partial_i\delta X^m)(\partial_j X^n)B_{mn}\right. \\
& \left.+(\tfrac{1}{2}G^{ij}\partial_i X^n\partial_j X^p g_{np,m}+\tfrac{1}{2}\theta^{ij}\partial_i X^n\partial_j X^p B_{np,m}-\Phi_{,m})\delta X^m-\theta^{ij}(\partial_i\delta A_j)\right),
\end{aligned}
$$

where we introduce the notation $G^{ij}\equiv M^{(ij)}$ and $\theta^{ij}\equiv M^{[ij]}$ (we symmetrize and antisymmetrize with unit strength).

The gauge field equation is

$$
J^j=\partial_i(e^{-\Phi}\sqrt{-M}\theta^{ij}), \tag{2.7}
$$

where $J^j\equiv\delta I_{WZ}/\delta A_j$ is the source current derived from varying the Wess-Zumino terms. The X^m field equation is

$$
\begin{aligned}
J_m = & -\partial_i\left(e^{-\Phi}\sqrt{-M}G^{ij}(\partial_j X^n)g_{mn}\right)-\partial_i\left(e^{-\Phi}\sqrt{-M}\theta^{ij}(\partial_j X^n)B_{mn}\right) \\
& +\sqrt{-M}\left(\tfrac{1}{2}(e^{-\Phi}G^{ij}\partial_i X^n\partial_j X^p g_{np,m}+e^{-\Phi}\theta^{ij}\partial_i X^n\partial_j X^p B_{np,m})-e^{-\Phi}\Phi_{,m}\right)
\end{aligned} \tag{2.8}
$$

where $J_m\equiv\delta I_{WZ}/\delta X^m$ denotes the contribution from the WZ terms in the action. We discuss the WZ contributions below.

To rewrite the equation in a natural covariant form we expand out the derivatives and use

$$
\begin{aligned}
\Gamma_{mnp} & \equiv \tfrac{1}{2}(g_{mn,p}+g_{mp,n}-g_{np,m}) \\
H_{mnp} & \equiv (B_{mn,p}+B_{np,m}+B_{pm,n}),
\end{aligned} \tag{2.9}
$$

where Γ_{mnp} is the Levi-Civita connection of the target space metric and H_{mnp} is the field strength of the NS-NS two form. Then we can express the field equation as

$$
\begin{aligned}
J_m = & -e^{-\Phi}\partial_i(\sqrt{-M}G^{ij})\partial_j X^n g_{mn}-\partial_i(e^{-\Phi}\sqrt{-M}\theta^{ij})\partial_j X^n B_{mn} \\
& -e^{-\Phi}\sqrt{-M}M^{ij}\left((\partial_i\partial_j X^n)g_{mn}+\tilde{\Gamma}_{mnp}\partial_i X^n\partial_j X^p\right), \\
& +e^{-\Phi}\sqrt{-M}\left(G^{ij}(\partial_i X^p\partial_j X^n)g_{mn}\Phi_{,p}-\Phi_{,m}\right)
\end{aligned} \tag{2.10}
$$

where we use symmetry to replace G^{ij} by M^{ij} in the first term of the second line and we introduce the torsionful connection $\tilde{\Gamma}=\Gamma-\frac{1}{2}H$.

The gauge field equation (2.7) can be used to substitute for the second term in the first line of (2.10) to give

$$
\begin{aligned}
J_m+J^j\partial_j X^n B_{mn} = & -e^{-\Phi}\partial_i(\sqrt{-M}G^{ij})\partial_j X^n g_{mn} \\
& -e^{-\Phi}\sqrt{-M}M^{ij}\left((\partial_i\partial_j X^n)g_{mn}+\tilde{\Gamma}_{mnp}\partial_i X^n\partial_j X^p\right) \\
& +e^{-\Phi}\sqrt{-M}\left(G^{ij}(\partial_i X^p\partial_j X^n)g_{mn}\Phi_{,p}-\Phi_{,m}\right).
\end{aligned} \tag{2.11}
$$

When $F_{ij} = B_{mn} = \Phi = 0$, the equation reduces to

$$J^m = -\sqrt{-g} g^{ij} \mathcal{K}^m_{ij} \tag{2.12}$$

where

$$\mathcal{K}^m_{ij} = \gamma^k_{ij} \partial_k X^m - (\partial_i \partial_j X^m) - \Gamma^m_{np} \partial_i X^n \partial_j X^p \tag{2.13}$$

is the second fundamental form (γ^k_{ij} is the Levi-Civita connection of the induced worldvolume metric). If in addition $J_m = 0$, the field equation becomes

$$g^{ij} \mathcal{K}^m_{ij} = 0, \tag{2.14}$$

that is, the trace of the second fundamental form of the embedding must be zero. For a flat target space, this condition is well-known; it is given in [38], for example.

Now let us derive the explicit form for the Wess-Zumino contributions. It is convenient to expand out the Wess-Zumino terms as

$$I_{WZ} = \sum_{n \geq 0} \frac{1}{n!(2!)^n q!} \int d^{p+1}\xi \epsilon^{i_1..i_{p+1}} \left((\mathcal{F})^n_{i_1..i_{2n}} C_{i_{2n+1}..i_{p+1}} \right), \tag{2.15}$$

where $\epsilon^{i_1 ... i_{p+1}}$ is the Levi-Civita tensor (with no metric factors) and $q = (p + 1 - 2n)$. Variation of the action then gives

$$\begin{aligned} \delta I_{WZ} \;\; = \;\; & \sum_{n \geq 0} \frac{1}{n!(2!)^n q!} \int d^{p+1}\xi \epsilon^{i_1 ... i_{p+1}} \left[n(2\partial_{i_1} \delta A_{i_2} - B_{mn,p} \delta X^p \partial_{i_1} X^m \partial_{i_2} X^n \right. \\ & -2B_{mn} \partial_{i_1} (\delta X^m) \partial_{i_2} X^n)(\mathcal{F})^{n-1}_{i_3..i_{2n}} C_{i_{2n+1} ... i_{p+1}} \\ & +(\mathcal{F})^n_{i_1..i_{2n}} \left((q C_{m_1..m_q} \partial_{i_{2n+1}} \delta X^{m_1} ... \partial_{i_{p+1}} X^{m_q} \right. \\ & \left. \left. + C_{m_1 ... m_q, m} \delta X^m \partial_{i_{2n+1}} X^{m_1} ... \partial_{i_{p+1}} X^{m_q}) \right] . \end{aligned} \tag{2.16}$$

This gives the following expression for the gauge field current appearing in (2.7)

$$J^{i_1} = \epsilon^{i_1..i_{p+1}} \sum_{n \geq 0} \frac{1}{n!(2!)^n (q-1)!} (\mathcal{F})^n_{i_2 ... i_{2n+1}} \bar{F}_{i_{2n+2} ... i_{p+1}}. \tag{2.17}$$

where[2]

$$\bar{F}_{m_1..m_{q+1}} = f_{m_1..m_{q+1}} - \frac{(q+1)!}{3!(q-2)!} H_{[m_1..m_3} C_{m_4..m_{q+1}]}. \tag{2.18}$$

In (2.17) we must sum over all possible values of n: in particular this means that for $p \geq 4$ we must include in the WZ term the dual RR potentials C_5, C_7 and C_9 (in type IIA) and

[2]Our convention for the field strengths is $f_{m_1 ... m_{q+1}} = (q+1)\partial_{[m_1} C_{m_2 ... m_{q+1}]}$.

C_6, C_8 (in type IIB)[3]. If we do not include the dual potentials there will remain gauge dependent terms in (2.17) involving $H_{[i_1..i_3}C_{i_4..i_{q+1}]}$.

From the X^m variation we find the expression for the current appearing in (2.8). The expression simplify when we consider the combination that enters in (2.11), and we obtain

$$J_m + J^j\partial_j X^n B_{mn} = \sum_{n\geq 0}\frac{1}{n!(2!)^n q!}\epsilon^{i_1..i_{p+1}}(\mathcal{F})^n_{i_1..i_{2n}}\bar{F}_{mi_{2n+1}...i_{p+1}}, \tag{2.20}$$

in which again we must include the dual RR potentials.

Let us summarise the D-brane field equations

$$\sum_{n\geq 0}\frac{1}{n!(2!)^n q!}\epsilon^{i_1..i_{p+1}}(\mathcal{F})^n_{i_1..i_{2n}}\bar{F}_{mi_{2n+1}...i_{p+1}} = e^{-\Phi}\left(\sqrt{-M}\left(G^{ij}\partial_i X^p\partial_j X^n g_{mn}\Phi_{,p} - \Phi_{,m}\right) - \mathcal{K}_m\right)$$

$$\partial_i(e^{-\Phi}\sqrt{-M}\theta^{ii_1}) = \epsilon^{i_1..i_{p+1}}\sum_{n\geq 0}\frac{1}{n!(2!)^n(q-1)!}(\mathcal{F})^n_{i_2...i_{2n+1}}\bar{F}_{i_{2n+2}...i_{p+1}}. \tag{2.21}$$

where

$$\mathcal{K}_m = -\partial_i(\sqrt{-M}G^{ij})\partial_j X^n g_{mn} - \sqrt{-M}M^{ij}\left((\partial_i\partial_j X^n)g_{mn} + \tilde{\Gamma}_{mnp}\partial_i X^n\partial_j X^p\right) \tag{2.22}$$

The gauge invariance of the field equation imply that $\mathcal{K}_m$ is gauge invariant. Furthermore, when $B_{mn} = 0$, $\mathcal{K}_m$ reduces to the trace of the second fundamental form. It follows that $\mathcal{K}_m$ is a gauge invariant generalization of the latter.

3 D5-brane embeddings in $AdS_5 \times S^5$

Let us now specialize to D5-brane embeddings in an $AdS_5 \times S^5$ background. The background geometry is

$$\begin{aligned} ds^2_{10} &= R^2\left(\frac{du^2}{u^2} + u^2(dx\cdot dx)_4 + ds^2_{S^5}\right); \\ ds^2_5 &= d\theta_1^2 + \sum_{k=2}^{5}\prod_{j=1}^{k-1}\sin\theta_j^2 d\theta_k^2; \\ f_5 &= 4R^4u^3 du\wedge dx^0\wedge dx^1\wedge dx^2\wedge dx^3 + 4R^4 d\Omega_5, \end{aligned} \tag{3.1}$$

[3]One defines the dual potentials as follows. The D7-brane in type IIB couples in (2.17) to the (pull back of the) 7-form field strength $\bar{F}_7$ which according to (2.18) satisfies (reverting to form notation),

$$\bar{F}_7 = dC_6 - H_3\wedge C_4. \tag{2.19}$$

This should be regarded as the defining equation for C_6, since we identify $\bar{F}_7$ with the (target space) dual of $\bar{F}_3$ and acting with the exterior derivative on (2.19) leads to the usual IIB field equation $d(*\bar{F}_3) = H_3\wedge f_5$. Analogous equations can be derived for the other dual potentials.

where the curvature is quantised as $R^4 = 4\pi g N (\alpha')^2$. Conventions for the IIB field equations are given in the appendix and since gN does not play a role here we will set $R = 1$ henceforth. Using (2.21) the D5-brane field equations reduce to

$$\begin{aligned} \partial_i(\sqrt{-M}\theta^{ii_1}) &= \frac{1}{5!}\epsilon^{i_1 i_2 i_3 i_4 i_5 i_6} f_{i_2 i_3 i_4 i_5 i_6}; \qquad (3.2) \\ \frac{1}{2!4!}\epsilon^{i_1 i_2 i_3 i_4 i_5 i_6} F_{i_1 i_2} f_{i_3 i_4 i_5 i_6 m} &= -\partial_i(\sqrt{-M} G^{ij}\partial_j X^n g_{mn}) + \tfrac{1}{2}\sqrt{-M}(G^{ij}\partial_i X^n \partial_j X^p g_{np,m}). \end{aligned}$$

where we remind our readers that $f_{i_3 i_4 i_5 i_6 m}$ denotes the pullback of f on the first four indices, i.e. $f_{i_3 i_4 i_5 i_6 m} = \partial_{i_3} X^{m_3} \partial_{i_4} X^{m_4} \partial_{i_5} X^{m_5} \partial_{i_6} X^{m_6} f_{m_3 m_4 m_5 m_6 m}$. The solution set of these equations describes all possible (including non-static) embeddings of D5-branes into the target space. Here we are interested in D5-branes which wrap an S^2 in the S^5 and whose remaining worldvolume directions preserve Poincaré invariance in three directions. Such embeddings can be found from the following ansatz: split the embedding coordinates X^m into $\{\xi^i, X^\lambda(\xi^i)\}$, where the worldvolume coordinates are

$$\xi^i = \{x^0, x^1, x^2, u, \theta_4, \theta_5\} \qquad (3.3)$$

and the transverse scalars are

$$X^\lambda = \{x^3(u) \equiv x(u), \theta_1, \theta_2, \theta_3\}, \qquad (3.4)$$

where for ease of notation we relabel x^3 as x and we assume that the only dependence of the transverse scalars on the worldvolume coordinates is in $x(u)$. We also switch on a worldvolume flux

$$F_{\theta_4 \theta_5} = q \sin\theta_4. \qquad (3.5)$$

With this ansatz it is straightforward to calculate all the quantities appearing in (3.2); for example,

$$\sqrt{-M} = u^2(1 + u^4 (x')^2)^{\frac{1}{2}} L_{\theta_\alpha} \sin\theta_4, \qquad (3.6)$$

where prime denotes the derivative with respect to u and

$$L_{\theta_\alpha} = (\prod_{\alpha=1}^{3} \sin^4\theta_\alpha + q^2)^{\frac{1}{2}}. \qquad (3.7)$$

Substituting the ansatz into (3.2), we find that the only independent equations are the ones deriving from the $X^m = \{x, \theta_1, \theta_2, \theta_3\}$ equations. The equation deriving from u follows from the x-equation, and the remaining equations are satisfied trivially. This is expected as worldvolume diffeomorphisms can be used to eliminate $p+1$ equations. The gauge field equation is satisfied automatically by the ansatz. The independent equations are

$$\begin{aligned} x &: \quad \partial_u \left(\frac{L_{\theta_\alpha}}{(1+u^4(x')^2)^{\frac{1}{2}}} u^6 x' - q u^4 \right) = 0; \\ \theta_\alpha &: \quad L_{\theta_\alpha}^{-1} (1+u^4(x')^2)^{\frac{1}{2}} \prod_{\beta\neq\alpha} \sin^4\theta_\beta \sin^3\theta_\alpha \cos\theta_\alpha = 0. \end{aligned} \tag{3.8}$$

To solve (3.8) we first note that the angular equations can be solved either when (i) all $\theta_\alpha = \frac{1}{2}\pi$, which we will refer to as a maximal sphere, or when (ii) one $\theta_\alpha = 0$ with the other two angles arbitrary, which we will refer to as a minimal sphere. The x equation in (3.8) yields

$$x' = \frac{(qu^4 - c)}{\sqrt{u^8 L_{\theta_\alpha}^2 - (qu^4 - c)^2}}, \tag{3.9}$$

where c is an integration constant. One can solve this differential equation using the first Appell hypergeometric functions of two variables, but we will not need this result. Note that there is an unbroken translational invariance in the x direction.

3.1 Branes wrapping maximal spheres

Let us now substitute solutions of the angular equations into (3.9). We focus first on the case of the brane wrapping a maximal sphere. In this case, (3.9) reduces to

$$x' = \frac{(qu^4 - c)}{u^2(u^8 + 2cqu^4 - c^2)^{\frac{1}{2}}}, \tag{3.10}$$

and the induced metric on the brane is

$$ds^2 = u^2(dx\cdot dx)_3 + \frac{u^6(1+q^2)}{(u^4 - u_+^4)(u^4 + u_-^4)} du^2 + (d\theta_4^2 + \sin^2\theta_4 d\theta_5^2), \tag{3.11}$$

where we write

$$(u^8 + 2cqu^4 - c^2) = (u^4 - u_+^4)(u^4 + u_-^4), \tag{3.12}$$

with $u_+^4, u_-^4 \geq 0$. The explicit form for the roots of (3.12) are

$$\begin{aligned} u_+^4 &= -cq + |c|\sqrt{1+q^2}; \\ u_-^4 &= cq + |c|\sqrt{1+q^2}. \end{aligned} \tag{3.13}$$

3.1.1 $AdS_4 \times S^2$ embeddings

The embedded geometry is $AdS_4 \times S^2$ when $c = 0$. In this limit (3.10) integrates to the simple expression

$$x = x_0 - \frac{q}{u}. \tag{3.14}$$

These embeddings were found by Karch and Randall [3]. Note that the $AdS_4 \times S^2$ embeddings exist even in the zero flux ($q = 0$) limit. The zero flux embedding must satisfy the zero extrinsic curvature trace condition (2.14) since for this solution J_m vanishes. It is a useful consistency check on our equations and solutions to calculate explicitly the extrinsic curvature for this embedding. The $\mathcal{K}^k_{ij}$ components vanish automatically since the extrinsic curvature can be projected onto the normal bundle. The $\mathcal{K}^x_{ij}$ components also vanish whilst for an S^2 embedded into S^5 we have

$$\mathcal{K}^{\theta_\alpha}_{44} = \mathcal{K}^{\theta_\alpha}_{55} \sin^{-2}\theta_4 = -\left[\sin^2\theta_2 \sin^2\theta_3 \sin\theta_1 \cos\theta_1, \sin^2\theta_3 \sin\theta_2 \cos\theta_2, \sin\theta_3 \cos\theta_3\right], \quad (3.15)$$

which implies that for this embedding the second fundamental form vanishes (the embedding is totally geodesic), a stronger condition than (2.14). This embedding has a very simple description as an intersection of a hyperplane with the hyperboloid representing AdS_5 in a flat ambient $6d$ spacetime with signature $(+,+,-,-,-,-)$. We will return to this topic in section 6.1.

3.1.2 Asymptotically $AdS_4 \times S^2$ embeddings

For the general solution in which $c \neq 0$ the induced geometry is asymptotically $AdS_4 \times S^2$ for $u \gg u_+$. Since from (3.10) $x(u)$ becomes imaginary for $u < u_+$ the brane ends at $u = u_+$ (x imaginary is not part of the target space). The induced metric does not have singular curvature at $u = u_+$: if we introduce a new coordinate $u = u_+ + \rho^2$ with $\rho \ll 1$, then the metric in this neighbourhood becomes

$$ds^2 = u_+^2 (dx \cdot dx)_3 + \frac{u_+^3(1+q^2)}{(u_+^4 + u_-^4)} d\rho^2 + (d\theta_4^2 + \sin^2\theta_4 d\theta_5^2), \quad (3.16)$$

which is non-singular at $\rho = 0$. Geodesics in the embedding geometry remain in the submanifold but they have finite endpoints at $u = u_+$; the embedded hypersurface is thus inextendible but incomplete.

One can bring the metric into a more conventional form by changing variables from u to the affine parameter U of the radial geodesic:

$$u^2 = \sqrt{U^{-4} - cq + \tfrac{1}{4}U^4 c^2 (1+q^2)}. \quad (3.17)$$

This brings the metric into the form[4]

$$ds^2 = (1+q^2)\frac{dU^2}{U^2} + \sqrt{U^{-4} - cq + \tfrac{1}{4}U^4 c^2(1+q^2)}(dx \cdot dx)_3 + (d\theta_4^2 + \sin^2\theta_4 d\theta_5^2). \quad (3.19)$$

[4]The metric (3.19) is mapped to itself under the inversion

$$U^4 \to \frac{4}{c^2(1+q^2)U^4} \quad (3.18)$$

and the range of U becomes $0 < U < U_+$. Thus one can complete the spacetime by extending the U

The range of U is

$$U_{+} = \left(\frac{2}{|c| \sqrt{1+q^2}} \right)^{\frac{1}{4}} \leq U < \infty. \tag{3.20}$$

In AdS/CFT the radial coordinate corresponds to the energy scale, which suggests that the dual theory develops a mass gap in the infrared. We will discuss the holographic interpretation of this embedding in section 5.

3.2 Branes wrapping minimal spheres: D5-branes collapsing to D3 branes

Let us now discuss embeddings in which the brane wraps a minimal sphere. In this case (3.9) reduces to

$$x' = \frac{(qu^4 - c)}{u^2 (2cqu^4 - c^2)^{\frac{1}{2}}}. \tag{3.21}$$

It is useful to rescale the parameter c such that $c = Cq$; this removes all q dependence in x':

$$x' = \frac{(u^4 - C)}{u^2 (2Cu^4 - C^2)^{\frac{1}{2}}}. \tag{3.22}$$

and the induced metric on the brane is then

$$ds^2 = u^2 (dx \cdot dx)_3 + \frac{u^6 du^2}{2C(u^4 - \frac{1}{2}C)}. \tag{3.23}$$

The embedding geometry is not asymptotically AdS_4: explicitly integrating (3.22) for $u >> 1$ we find that

$$x \sim x_0 + \frac{1}{\sqrt{2C}} u. \tag{3.24}$$

This implies that the defect in the dual field theory is located at $x \to \infty$. Since x' becomes imaginary for $u^4 < \frac{1}{2}C = u_c^4$, the brane ends at u_c. The induced geometry is non-singular at u_c though the embedded hypersurface is again incomplete.

Note that the induced metric on the S^2 is degenerate. However, provided that the flux through the sphere is non-zero M_{ij} is non-degenerate and hence invertible. Since the field equations were derived in section 1 assuming the invertibility of M_{ij}, but without assuming that g_{ij} is non-degenerate, these embeddings are admissible solutions of the field equations provided that q is non-zero. Examining the worldvolume action one finds that the parameter q appears only as an overall parameter and can thus be scaled to plus or minus one.

Physically there is a very natural interpretation of embeddings in which the S^2 is minimal. The D5-brane has effectively collapsed to a D3-brane embedded in AdS_5 and in fact

variable to range from zero to infinity. The resulting manifold covers twice the original allowed region. This completion corresponds to reflective boundary conditions for the geodesics that originally terminated at $u = u_{+}$.

this degenerate D5-brane embedding can be found as a solution of the D3-brane equations of motion. If the flux is positive we get a D3-brane, whereas negative flux corresponds to anti-D3-branes. To show this, let us look for D3-brane embeddings which preserve a $(2+1)$-dimensional Poincaré invariance and which lie at a point in the S^5.

The D3-brane field equations in the $AdS_5 \times S^5$ target space are

$$\frac{1}{4!}\epsilon^{i_1 i_2 i_3 i_4} f_{i_1 i_2 i_3 i_4 m} = -\partial_i(\sqrt{-M} G^{ij} \partial_j X^n g_{mn}) + \tfrac{1}{2}\sqrt{-M}(G^{ij}\partial_i X^n \partial_j X^p g_{np,m}). \qquad (3.25)$$

An appropriate ansatz for worldvolume coordinates is

$$\xi^i = \{x^0, x^1, x^2, u\}, \qquad (3.26)$$

whilst the transverse scalars are

$$X^\lambda = \{x(u), \theta_a\}. \qquad (3.27)$$

Then the only equation of motion (coming from the u and x field equations, which are equivalent) is

$$\partial_u \left(\frac{u^6 x'}{\sqrt{1 + u^4 (x')^2}} - u^4 \right) = 0. \qquad (3.28)$$

The (constant) angles on the S^5 are arbitrary. Since the general solution of (3.28) is (3.22), this implies that the collapsed D5-brane wrapping a minimal 2-sphere can be interpreted as a D3-brane.

There are no special limits of (3.22) when $C = 0$ or $q = 0$. We cannot solve the equations of motion for $C = 0$ even when $q \neq 0$. In the $q \to 0$ limit, M_{ij} is totally degenerate along the S^2 directions and the solution is not admissible.

We will see in section 4.3 that the embedding discussed above is not supersymmetric. The reason is that the collapsed D5-brane is a D3-brane which is misaligned with respect to D3-branes that create the background. Such branes would be expected to have an instability that tends to rotate them to become aligned with the D3-branes creating the AdS background. The ansatz (3.3) and (3.4) used so far is not appropriate for finding such configurations. The appropriate ansatz describing D5-branes wrapping the S^2 and whose worldvolumes lie along x is

$$\begin{aligned} \xi^i &= \{x^0, x^1, x^2, x, \theta_4, \theta_5\}; \qquad (3.29) \\ X^\lambda &= \{u, \theta_1, \theta_2, \theta_3\}; \\ F_{\theta_4\theta_5} &= q \sin\theta_4, \end{aligned}$$

where all transverse scalars are constant. The only field equations which are not already satisfied by the ansatz are

$$\begin{aligned} u &: \quad u^3(L_{\theta_\alpha} - q) = 0; \\ \theta_\alpha &: \quad u^4 L_{\theta_\alpha}^{-1} \prod_{\beta \neq \alpha} \sin^4\theta_\beta \sin^3\theta_\alpha \cos\theta_\alpha = 0. \end{aligned} \tag{3.30}$$

The only solution for which M_{ij} is non-degenerate is an S^2 minimal solution with non-zero flux q for any u_0. As before, one can scale $q = \pm 1$. We will see later that the solution with $q = 1$ preserves 1/2 supersymmetry and can be interpreted as a supersymmetric D3-brane whereas $q = -1$ breaks all supersymmetries and corresponds to an anti-D3-brane.

4 Supersymmetry of embeddings

Associated with every brane embedding is a kappa symmetry projection which is defined (using quantities appearing in (2.1)) as [39, 40, 41, 42, 43, 44]

$$d^{p+1}\xi\Gamma = -e^{-\Phi}\mathcal{L}_{DBI}^{-1} e^{\mathcal{F}} \wedge X|_{vol}, \tag{4.1}$$

with

$$X = \bigoplus_n \Gamma_{(2n)} K^n I, \tag{4.2}$$

where $|_{vol}$ indicates that one should pick the terms proportional to the volume form, and the operations I and K act on spinors as $I\psi = -i\psi$ and $K\psi = \psi^*$. $\mathcal{L}_{DBI}^{-1}$ is the value of the DBI Lagrangian evaluated on the background. Here we have used the notation

$$\Gamma_{(n)} = \frac{1}{n!} d\xi_n^i \wedge ... \wedge d\xi^{i_1} \Gamma_{i_1...i_n}, \tag{4.3}$$

where $\Gamma_{i_1...i_n}$ is the pullback for the target space gamma matrices

$$\Gamma_{i_1...i_n} = \partial_{i_1} X^{m_1} ... \partial_{i_n} X^{m_n} \Gamma_{m_1...m_n} \tag{4.4}$$

It has been shown in [39, 40, 41, 42, 43, 44] that Γ squares to one and is traceless. It follows that one can use Γ to project out half of the worldvolume fermions, thus equating the worldvolume fermionic and bosonic degrees of freedom.

A given brane embedding within a supersymmetric target background preserves some fraction of the supersymmetry provided that the Killing spinors of the background ϵ are consistent with the projection

$$\Gamma\epsilon = \epsilon. \tag{4.5}$$

In other words the restriction of the Killing spinors on the worldvolume should satisfy (4.5). We note here that (given our choice of conventions) we will need to choose the positive sign in (4.5) in the AdS embedding, i.e. there are no supersymmetric D5-embeddings on $AdS_5 \times S^5$ with the negative sign.

To proceed we need the explicit form of the Killing spinors of the background. The $AdS_5 \times S^5$ background geometry preserves maximal supersymmetry since the dilatino equation is trivially satisfied and the gravitino equation

$$\left(D_m + \tfrac{1}{2} i\gamma^{01234}\Gamma_m\right)\epsilon = 0, \tag{4.6}$$

admits a full compliment of thirty-two independent solutions. (Conventions for the supersymmetry variations are given in the appendix.) We denote by $\gamma_a = e_a^m \Gamma_m$ the tangent space gamma matrices. For the case at hand, they are given by

$$\gamma_p = \frac{1}{u}\Gamma_p,\ (p=0,1,2,3), \quad \gamma_4 = u\Gamma_u, \quad g_a = \left(\prod_{j=1}^{a-5} \frac{1}{\sin\theta_j}\right)\Gamma_{\theta_{a-4}}\ (a=5,6,7,8,9) \tag{4.7}$$

(when $a = 5$ the product is equal to one).

Following closely Claus and Kallosh [45], we now solve for the explicit form of the Killing spinors. It is convenient to introduce the projections:

$$\epsilon_\pm = \mathcal{P}_\pm \epsilon = \tfrac{1}{2}(1 \mp \gamma^{0123} I)\epsilon = \tfrac{1}{2}(1 \pm i\gamma^{0123})\epsilon. \tag{4.8}$$

Using these projectors we can rewrite the Killing spinor equations (4.6) as

$$\begin{aligned} \partial_p\epsilon_- + u\gamma_p\gamma_4\epsilon_+ &= 0; \\ \partial_p\epsilon_+ &= 0; \\ \partial_u\epsilon_\pm \pm \tfrac{1}{2}u^{-1}\epsilon_\pm &= 0; \\ D_a\epsilon_\pm \pm \tfrac{1}{2}\gamma_4\Gamma_a\epsilon_\pm &= 0, \end{aligned} \tag{4.9}$$

where D_a is the covariant derivative on the sphere. The full solution to the Killing spinor equation is the combination $\epsilon = \epsilon_+ + \epsilon_-$ with

$$\begin{aligned} \epsilon_+ &= -u^{-\frac{1}{2}}\gamma_4 h(\theta_a)\eta_2; \\ \epsilon_- &= u^{\frac{1}{2}} h(\theta_a)(\eta_1 + x\cdot\gamma\eta_2), \end{aligned} \tag{4.10}$$

where η_1 and η_2 are constant spinors, satisfying

$$\eta_1 = \mathcal{P}_-\eta_1, \qquad \eta_2 = \mathcal{P}_+\eta_2. \tag{4.11}$$

η_1 and η_2 are complex spinors of negative and positive chirality respectively so this gives us the 32 independent real spinors. That is, we can choose

$$\begin{aligned} \eta_1 &= \lambda - i\gamma^{0123}\lambda; \\ \eta_2 &= \eta + i\gamma^{0123}\eta, \end{aligned} \tag{4.12}$$

where λ and η are real spinors of negative and positive chirality respectively, with 16 independent components. Such a choice makes the complex conjugation of the spinors manifest. The function $h(\theta_a)$ appearing in both spinors results from the Killing equation on the sphere and is given by

$$h(\theta_a) = \exp(\tfrac{1}{2}\theta_1\gamma_{45})\exp(\tfrac{1}{2}\theta_2\gamma_{56})\exp(\tfrac{1}{2}\theta_3\gamma_{67})\exp(\tfrac{1}{2}\theta_4\gamma_{78})\exp(\tfrac{1}{2}\theta_5\gamma_{89}). \tag{4.13}$$

Explicit forms for the Killing spinors of $AdS_5 \times S^5$ appeared previously in, for example, [46] and [45].

4.1 Supersymmetry of asymptotically $AdS_4 \times S^2$ branes

We would now like to check whether the asymptotically $AdS_4 \times S^2$ brane embedding found in the previous section preserves supersymmetry. The explicit form of the kappa symmetry projection is

$$\epsilon = \frac{i}{u^4(1+q^2)}\gamma^{012}\left((qu^4 - c)\gamma^3 + (u^8 + 2cqu^4 - c^2)^{\frac{1}{2}}\gamma^4\right)\left(\gamma^{89}\epsilon^* - q\epsilon\right). \tag{4.14}$$

For reasons discussed in Appendix B, we choose to work with the projector Γ involving the flux rather than to use a similarity transformation to obtain a projector not involving the flux Γ' as in [44]. The difference with cases considered previously is that in our case the worldvolume embedding depends explicitly on the flux. Preservation of supersymmetry requires that this condition must be satisfied for some subset of the background Killing spinors at all points on the brane worldvolume. In particular, it must hold at all values of $x^p = (x^0, x^1, x^2)$. From the terms in the Killing spinors which are linear in x^p we find the following condition

$$\begin{aligned} \gamma^p(1 + i\gamma^{0123})h(\theta_a)\eta &= \frac{i\gamma^{012}}{u^4(1+q^2)}\left((qu^4 - c)\gamma^3 + (u^8 + 2cqu^4 - c^2)^{\frac{1}{2}}\gamma^4\right) \\ &\quad \times \left(\gamma^{89p}(1 - i\gamma^{0123}) - q\gamma^p(1 + i\gamma^{0123})\right)h(\theta_a)\eta. \end{aligned} \tag{4.15}$$

Recalling that η is real, this can be separated into two conditions coming from the real and imaginary parts:

$$\begin{aligned} h(\theta_a)\eta &= \frac{1}{u^4(1+q^2)}\left((qu^4 - c) - (u^8 + 2cqu^4 - c^2)^{\frac{1}{2}}\gamma^{34}\right)(\gamma^{89} + q)h(\theta_a)\eta; \\ h(\theta_a)\eta &= -\frac{1}{u^4(1+q^2)}\left((qu^4 - c) + (u^8 + 2cqu^4 - c^2)^{\frac{1}{2}}\gamma^{34}\right)(\gamma^{89} - q)h(\theta_a)\eta, \end{aligned} \tag{4.16}$$

which in turn imply the constraints

$$\begin{aligned}\left((qu^4 - c) + q(u^8 + 2cqu^4 - c^2)^{\frac{1}{2}}\gamma^{3489}\right) h(\theta_a)\eta &= 0; \\ \frac{c}{qu^4} h(\theta_a)\eta &= 0.\end{aligned} \tag{4.17}$$

In this section we consider the case of non-zero c: then these constraints can manifestly not be satisfied for non-zero η. So setting $\eta = 0$ let us impose the kappa symmetry projection on the remaining parts of the Killing spinors involving λ. This implies the conditions

$$\begin{aligned} h(\theta_a)\lambda &= \frac{1}{u^4(1+q^2)}\left((qu^4 - c) - (u^8 + 2cqu^4 - c^2)^{\frac{1}{2}}\gamma^{34}\right)(\gamma^{89} + q)h(\theta_a)\lambda; \\ h(\theta_a)\lambda &= -\frac{1}{u^4(1+q^2)}\left((qu^4 - c) + (u^8 + 2cqu^4 - c^2)^{\frac{1}{2}}\gamma^{34}\right)(\gamma^{89} - q)h(\theta_a)\lambda, \end{aligned} \tag{4.18}$$

which impose constraints on λ identical to those in (4.17). Thus λ can also not be non-zero for non-zero c and the embeddings with $c \neq 0$ break all the supersymmetry.

4.2 Supersymmetry of $AdS_4 \times S^2$ branes

When $c = 0$ the kappa symmetry projection is given by

$$\epsilon = \frac{i}{(1+q^2)}\gamma^{012}(q\gamma^3 + \gamma^4)(\gamma^{89}\epsilon^* - q\epsilon) \tag{4.19}$$

The restriction of the Killing spinors to the worldvolume is

$$\epsilon = -u^{-1/2}(\gamma^4 + q\gamma^3)h(\theta_a)\eta_2 + u^{1/2}h(\theta_a)(x_0\gamma^3\eta_2 + \eta_1) + u^{1/2}h(\theta_a)x^p\gamma_p\eta_2 \tag{4.20}$$

The analysis of the previous section leading to (4.17) still holds but since we now take $c = 0$ the second condition in (4.17) is trivially satisfied and the first condition reduces to

$$(1 + \gamma^{3489})h(\theta_a)\eta = 0, \tag{4.21}$$

Since $h(\theta_a)$ is invertible, (4.21) implies that half of the η spinors are projected out. To explicitly obtain the projection on the spinor η, we multiply (4.21) by $h(\theta_a)^{-1}$ and compute $h(\theta_a)^{-1}\gamma^{3489}h(\theta_a)$. This can be done effectively by using repeatedly identities of the form

$$e^{-\frac{1}{2}\theta\gamma_{p(p+1)}}\gamma_{(p+1)q}e^{\frac{1}{2}\theta\gamma_{p(p+1)}} = \cos\theta + \gamma_{qp}\sin\theta. \tag{4.22}$$

Recalling that $\theta_1 = \theta_2 = \theta_3 = \pi/2$ on the worldvolume, we finally obtain

$$(1 + \gamma^{3789})\eta = 0. \tag{4.23}$$

Let $\gamma^{3789}\eta_\pm = \pm\eta_\pm$. Equation (4.23) eliminates the η_+ spinors.

We have just shown that the parts of the projection condition (4.19) involving terms linear in x^p can be satisfied with q arbitrary, provided we impose a projection onto the constant spinor η. The projection condition (4.19) must be satisfied at all points on the worldvolume, namely at all values of u. This means that the projection holds independently for terms proportional to $u^{\frac{1}{2}}$ and terms proportional to $u^{-\frac{1}{2}}$ in (4.19). The latter condition is automatically satisfied when (4.21) holds. Terms proportional to $u^{\frac{1}{2}}$ in (4.19) imply

$$(1+\gamma^{3489})h(\theta_a)\lambda = -2\gamma^3 x_0 h(\theta_a)\eta, \tag{4.24}$$

or equivalently

$$(1+\gamma^{3789})\lambda = -2\gamma^3 x_0 \eta. \tag{4.25}$$

Let $\gamma^{3789}\lambda_\pm = \pm\lambda_\pm$. Equation (4.25) determines λ_+ in terms of η_-, but leaves undetermined λ_-.

Putting together all the projection conditions we see that in total sixteen of the Killing spinors are preserved by the embedding and one half of the target space supersymmetry is broken. Note that the projections on the constant spinors do not depend on the flux. They do however depend on the asymptotic value of $x = x_0$. Probe branes with different values of x_0 (different locations of the defect in the boundary theory) will preserve the same η spinors but different λ spinors. Thus two or more defects in the boundary theory will break the supersymmetry from one half to one quarter. It would be interesting to understand this from the perspective of the defect conformal field theory.

4.3 Supersymmetry of branes wrapping minimal spheres

The kappa symmetry projection for the brane wrapping a minimal sphere is

$$\epsilon = -i\gamma^{012}\left((1-\frac{C}{u^4})\gamma^3 + \frac{1}{u^4}(2Cu^4 - C^2)^{\frac{1}{2}}\gamma^4\right)\epsilon. \tag{4.26}$$

Preservation of supersymmetry requires that this condition must be satisfies for some subset of the background Killing spinors at all points on the brane worldvolume. From the terms in the Killing spinors which are linear in x^p we find that

$$\gamma^p(1+i\gamma^{0123})h(\theta_a)\eta = -i\gamma^{012}\left((1-\frac{C}{u^4})\gamma^3 + \frac{1}{u^4}(2Cu^4 - C^2)^{\frac{1}{2}}\gamma^4\right)\gamma^p(1+i\gamma^{0123})h(\theta_a)\eta, \tag{4.27}$$

which can be separated into real and imaginary parts

$$\begin{aligned} h(\theta_a)\eta &= \left((1-\frac{C}{u^4})\gamma^3 - \frac{1}{u^4}(2Cu^4 - C^2)^{\frac{1}{2}}\gamma^{34}\right)h(\theta_a)\eta; \qquad (4.28)\\ h(\theta_a)\eta &= \left((1-\frac{C}{u^4})\gamma^3 + \frac{1}{u^4}(2Cu^4 - C^2)^{\frac{1}{2}}\gamma^{34}\right)h(\theta_a)\eta;, \end{aligned}$$

which can only be satisfied if

$$\frac{C}{u^4} h(\theta_a)\eta = 0; \;\; \gamma^{34}(2Cu^4 - C^2)^{\frac{1}{2}} h(\theta_a)\eta = 0, \tag{4.29}$$

conditions which do not have non-zero solutions η when C is non-zero (and recall that C cannot be zero). Thus we must set $\eta = 0$. Imposing the kappa symmetry projection on the remaining parts of the Killing spinors involving λ we find constraints on λ identical to those in (4.29). Thus there are no non-zero solutions to the kappa symmetry projection and these embeddings break all the supersymmetry.

Finally we discuss the D5-brane embedding described in (3.29). The kappa symmetry projection reads

$$\epsilon = -isgn(q)\gamma^{0123}\epsilon \tag{4.30}$$

where $sgn(q)$ is the sign of flux (which as we argued can be scaled to ± 1). This equation should hold at all points on the worldvolume. Inserting the Killing spinors from (4.10) and examining the terms linear in x^p we find that the resulting equation is satisfied identically when $sqn(q) = 1$, but it projects out the η spinor when $sqn(q) = -1$. The remaining equations project out the η spinor when $sqn(q) = 1$, and the λ spinor when $sqn(q) = -1$. We conclude that the embedding with $sqn(q) = 1$ preserves half of supersymmetry and can be identified with a supersymmetric D3-brane whilst the embedding with $sqn(q) = -1$ breaks all supersymmetry and corresponds to an anti-D3 brane.

5 Dual interpretation: RG flows on the defect

D5-branes wrapping submanifolds of $AdS_5 \times S^5$ may be viewed as the near-horizon limit of intersecting D3-D5 systems. The AdS/CFT duality is considered to act twice: both in the bulk and on the worldvolume of the D5-brane. The dual field theory could be obtained directly by considering the intersections of the D3-brane and D5-brane worldvolume theories: in the case we discuss here it will be $\mathcal{N} = 4$ SYM theory coupled to a three dimensional defect. The defect theory may be associated with the boundary of the AdS_4 of the $AdS_4 \times S^2$ D5-brane and as such it should be a conformal field theory. The defect theory contains both ambient fields, which follow from the $\mathcal{N} = 4$ SYM, and fields confined to the defect. An explicit construction of this defect CFT theory was given recently in [4]; see also [8].

Within the approximations used in this paper, the D5-brane theory does not backreact on the bulk. This means that we can consider deformations in which the boundary theory remains conformal but the defect theory runs. This is precisely what is happening in our asymptotically $AdS_4 \times S^2$ embeddings into the $AdS_5 \times S^5$ background.

Since in our embeddings only the AdS_4 part of the solution is deformed this implies that the defect QFT still has the same R-symmetry as the defect CFT. Using the operator-field dictionary developed in [4] and the asymptotic form of the worldvolume fields we can show that the RG-flow corresponds to a vev deformation of the defect theory. To see this, note that the active scalar in our embeddings behaves for large u as

$$ux \sim ux_0 + \frac{c}{5u^4}, \tag{5.1}$$

where we assume that $q = 0$ (since this is the only case considered in [4]). Using the standard AdS/CFT dictionary [47], [48], [49], [50], this suggests that this scalar is dual to an operator in the defect theory of conformal dimension four.

The identification between the scalar ux and such an operator was made in [4], where it was shown that the operator in question is a certain four supercharge descendant of the second floor chiral primary on the defect. Let us briefly summarise their arguments; for more details we refer to [4].

The defect theory has an $SU(2)_H \times SU(2)_V$ R-symmetry; the first factor is associated with rotations of the S^2 wrapped by the D5-brane whilst the second is associated with the symmetry of transverse directions. The ambient fields on the defect follow from the decomposition of the $\mathcal{N} = 4$ vector multiplet into a 3d $\mathcal{N} = 4$ vector multiplet and a 3d $\mathcal{N} = 4$ adjoint hypermultiplet. The bosonic components of the former are (A_k, X_V^A) and of the latter are (A, X_H^I), where we use $k = 0, 1, 2$ to denote spacetime indices, A denotes a vector index of $SU(2)_V$ and I denotes a vector index of $SU(2)_H$.

There is also a $d = 3$ hypermultiplet on the defect which transforms in the fundamental of the $SU(N)$ gauge group. This consists of an $SU(2)_H$ doublet of complex scalars q^m and an $SU(2)_V$ doublet of Dirac fermions ψ^i. It was argued in [4] that the lowest chiral primary of the defect theory is the triplet

$$C^I \equiv \bar{q}^m \sigma^I_{mn} q^n. \tag{5.2}$$

T-duality arguments were then used to show that higher chiral primaries must arise from taking the symmetrised traceless part of this operator with the scalars from the ambient hypermultiplet, namely

$$C^{I_1..I_l} = C^{(I_1} X_H^{I_2} ... X_H^{I_l)}. \tag{5.3}$$

Now the $l = 2$ chiral primary has $\Delta = 2$; thus it must have a four supercharge descendant $\mathcal{O}_x$ which has $\Delta = 4$ and is an R-singlet.

Now the AdS/CFT dictionary tells us that we should regard x_0 as a source for the operator $\mathcal{O}_x$, and c as its expectation value. Since the operator that gets the vev is a specific scalar component of a $3d$ superfield, the vev breaks supersymmetry, as we found. Furthermore, the operator is an R-singlet in accordance with the fact that the S^2 part of the solution is undeformed. Thus our results provide evidence for the dictionary proposed in [4].

An interesting feature of this RG-flow is that the theory develops a mass gap in the infrared. We will now suggest a way to understand this, based on extending our embeddings to embeddings of a probe D5-brane in the full D3-brane background. Let us write the metric in the D3-brane background as

$$ds^2 = f(r)^{-\frac{1}{2}}(dx \cdot dx)_4 + f(r)^{\frac{1}{2}}(dr^2 + r^2 d\Omega_5^2), \tag{5.4}$$

where $f(r) = (1 + R^4/r^4)$. Then the N D3-branes are located at the origin in the transverse space. Standard intersection rules tell us that a probe D5-brane intersecting the D3-branes on a membrane will preserve supersymmetry. This corresponds to taking the worldvolume directions of the D5-brane to be $\{x^0, x^1, x^2, r, \Omega_2\}$, where the brane wraps a maximal S^2 in the S^5 and all other transverse scalars, including $x^3 \equiv x$, are constant. Taking the near horizon limit, $r \ll R$, reproduces the $AdS_4 \times S^2$ embeddings considered here. The probe D5-brane intersects the D3-branes on a submanifold on which $r = 0$ and $x = x_0$; this means that there are massless 3-5 strings.

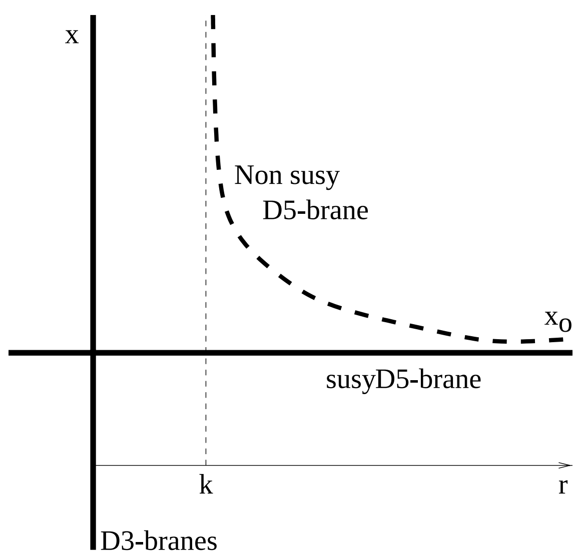

Figure 1: Probe D5-branes in the D3-brane background

Suppose we now look for an embedding in which x depends explicitly on r; the analysis follows closely that of the section three and leads to a defining equation for $x(r)$:

$$0 = \frac{\partial}{\partial r}\left(\frac{x'(r) f^{-1}(r)}{\sqrt{(1 + f(r)^{-1}(x'(r))^2)}}\right), \tag{5.5}$$

which in the near horizon limit reproduces (3.10). Using the explicit form of the Killing spinors in the D3-brane background one can show that such an embedding breaks supersymmetry unless x is constant (as expected from our near horizon analysis). Furthermore, examining the asymptotics of the general solution to (5.5), we find that the schematic dependence of $x(r)$ is as illustrated in Figure 1.

The key point is that $r \geq k$ (where $k \geq 0$ corresponds to the c appearing in (3.10)) and so the non-susy probe D5-brane does not intersect the D3-branes: k measures the separation of the probe from the D3-branes. All 3-5 strings are massive and this is the origin of the mass gap in the defect quantum field theory. This is consistent with the fact that the operator $\mathcal{O}_x$, which we argued gets a vev, contains fields of the defect hypermultiplet.

It would be interesting to further explore this holographic duality by computing correlation functions and Wilson loops [51, 52]. To properly compute correlation functions one would need to implement the program of holographic renormalization [53, 54, 55] in the current setting. We leave this problem for future work.

6 Penrose limits

In this section we consider the Penrose limit of $AdS_5 \times S^5$ leading to a pp-wave and the limit thus induced on the $AdS_4 \times S^2$ brane embeddings.

6.1 Embeddings in global coordinates

As is well-known, AdS_5 can be described as a pseudosphere embedded in a 6-dimensional ambient space. Introducing coordinates Y^μ for this ambient space, then

$$(Y^0)^2 + (Y^1)^2 - (Y^2)^2 - (Y^3)^2 - (Y^4)^2 - (Y^5)^2 = R^2, \tag{6.1}$$

where R is the curvature of the AdS_5 hypersurface. Global coordinates for AdS_5 are related to the Cartesian coordinates Y^μ as

$$\begin{aligned} Y^0 &= R\cosh\rho\cos\tau; & Y^1 &= R\cosh\rho\sin\tau; \\ Y^2 &= R\sinh\rho\cos\chi\sin\psi; & Y^3 &= R\sinh\rho\cos\chi\cos\psi; \\ Y^4 &= R\sinh\rho\sin\chi\sin\phi; & Y^5 &= R\sinh\rho\sin\chi\cos\phi, \end{aligned} \tag{6.2}$$

and the AdS_5 metric in these coordinates is

$$ds_5^2 = R^2 \left[-\cosh^2 \rho d\tau^2 + d\rho^2 + \sinh^2 \rho (d\chi^2 + \cos^2 \chi d\psi^2 + \sin^2 \chi d\phi^2) \right]. \tag{6.3}$$

We could now repeat the analysis of the previous sections to find $AdS_4 \times S^2$ embeddings in this coordinate system, by solving the D-brane equations of motion explicitly. It is convenient however to use a different approach. We found in the Poincaré coordinate system that the supersymmetric branes wrapped an AdS_4 submanifold, of curvature radius $R\sqrt{1+q^2}$ where q is the charge on the S^2. This should be a coordinate independent statement.

To find such AdS_4 submanifolds in global coordinates it is most convenient to start from the Cartesian embedding coordinates [5]. Suppose we choose the codimension one hypersurface $Y^2 = Rq$ in the ambient space; then inserting this condition into (6.1) we find the intersection of this hypersurface with the AdS_5 hypersurface is a 4-dimensional hypersurface satisfying

$$(Y^0)^2 + (Y^1)^2 - (Y^3)^2 - (Y^4)^2 - (Y^5)^2 = R^2(1+q^2). \tag{6.4}$$

This implies that the 4-dimensional hypersurface is also AdS, with curvature radius $R\sqrt{1+q^2}$. Other AdS_4 submanifolds can be obtained by choosing different codimension one hypersurfaces; the submanifolds are related to each other by the action of the five-dimensional isometry group $SO(4,2)$.

The induced metric on the hypersurface $Y^2 = Rq$ in AdS_5 can be written in terms of the global coordinates as

$$\begin{aligned} ds_4^2 &= R^2 \left(-\cosh^2 \rho d\tau^2 + d\rho^2 + \sinh^2 \rho (d\chi^2 + \sin^2 \chi d\phi^2) \right) \\ &\quad + \frac{R^2 q^2}{(\sinh^2 \rho \cos^2 \chi - q^2)} (\sinh \rho \sin \chi d\chi - \cosh \rho \cos \chi d\rho)^2, \end{aligned} \tag{6.5}$$

where we eliminate ψ in favour of (ρ, χ).

This submanifold defines a brane embedding in which the worldvolume coordinates of the AdS_4 are (τ, ρ, χ, ϕ) with ψ a transverse scalar given by

$$\psi = \arcsin\left(\frac{q}{\sinh \rho \cos \chi}\right). \tag{6.6}$$

Note that $\psi = 0$ for zero flux embeddings ($q = 0$) and that $\psi \to 0$ as $\rho \to \infty$ for finite flux embeddings.

[5] This approach was also used in [1] to find AdS_2 submanifolds of AdS_3 in different coordinate systems.

Having found an AdS_4 embedding in global coordinates by this quick route, one can then verify that it does indeed satisfy the D5-brane equations of motion, with the D5-brane wrapping a maximal S^2 with flux q. Furthermore, one can check explicitly that the curvature scalar of (6.5) is indeed

$$\mathcal{R} = \frac{12}{R^2(1+q^2)}. \tag{6.7}$$

One could also use an explicit relationship between Poincaré and global coordinates to map this embedding to that found previously.

6.2 Penrose limits

The reason for switching to global coordinates is that there is then a particularly simple Penrose limit taking $AdS_5 \times S^5$ to an Hpp-wave. The background is

$$\begin{aligned} ds^2_{10} &= ds^2(AdS_5) + R^2\left(d\theta_1^2 + \sum_{k=2}^{5}\prod_{j=1}^{k-1}\sin\theta_j^2 d\theta_k^2\right); \\ f_5 &= 4R^4\sinh^3\rho\cosh\rho d\tau\wedge d\rho\wedge d\Omega_3 + 4R^4 d\Omega_5, \end{aligned} \tag{6.8}$$

where the AdS_5 metric is given in (6.3). A Penrose limit focuses on the geometry in the neighbourhood of a null geodesic [29], [30]. We consider here limits arising from a particle moving in the θ_5 direction, sitting at $\theta_a = \frac{1}{2}\pi$ for $a \neq 5$. Let us introduce coordinates

$$\theta_a = \tfrac{1}{2}\pi - \frac{z^a}{R}, \tag{6.9}$$

for $a \neq 5$; the Penrose limit requires $R \to \infty$ so expanding the metric on the S^5 for large R we find

$$ds_5^2 = (dz^a)^2 + (R^2 - (z^a)^2)d\theta_5^2 + ..., \tag{6.10}$$

where the ellipses denotes terms in negative powers of R. Introducing coordinates

$$x^+ = \tfrac{1}{2}(\tau + \theta_5);\;\; x^- = \tfrac{1}{2}R^2(\tau - \theta_5);\;\; \rho = \frac{r}{R}, \tag{6.11}$$

and then taking the limit $R \to \infty$ the target space metric becomes [9], [11]

$$ds^2 = -4(dx^+dx^-) - ((y^a)^2 + (z^a)^2)(dx^+)^2 + (dz^a)^2 + (dy^a)^2 \tag{6.12}$$

which is a plane wave metric, whilst the flux becomes

$$f_{+y^1y^2y^3y^4} = f_{+z^1z^2z^3z^4} = 4, \tag{6.13}$$

where we introduce for convenience below Cartesian coordinates such that $y^a y^a = r^2$,

$$y^1 = r\sin\chi\cos\phi, \quad y^2 = r\sin\chi\sin\phi, \quad y^3 = r\cos\chi\cos\psi, \quad y^4 = r\cos\chi\sin\psi. \tag{6.14}$$

Now let us consider the Penrose limit applied to the worldvolume fields. The induced worldvolume metric is

$$ds_6^2 = ds_4^2 + R^2(d\theta_4^2 + \sin^2\theta_4 d\theta_5^2), \tag{6.15}$$

where the AdS_4 metric is given in (6.5). There is also a flux on the S^2, $F_{\theta_4\theta_5} = q\sin\theta_4$. The Penrose scaling of the induced worldvolume metric is implicitly defined given the scaling of the target space metric [9], [10].

The scaling of the worldvolume flux is determined by requiring that any solution of the D-brane field equations remains a solution in the Penrose limit. As discussed in [10] this means that the flux on the brane in the Penrose limit $\tilde{F}$ is related to the original flux F by $\tilde{F} = R^2 F$. Since in the Penrose limit

$$F \rightarrow \frac{q}{R} dz^4 \wedge d\theta_5, \tag{6.16}$$

a finite value for $\tilde{F}$ requires that we set $\tilde{q} = qR$.

Then applying this Penrose limit to the induced worldvolume metric in (6.5) we get

$$\begin{aligned} ds_6^2 &= -4dx^+dx^- - ((y^1)^2 + (y^2)^2 + (y^3)^2 + \tilde{q}^2 + (z^4)^2)(dx^+)^2 \\ &\quad +(dz^4)^2 + (dy^1)^2 + (dy^2)^2 + (dy^3)^2, \end{aligned} \tag{6.17}$$

where the y^i coordinates are defined in (6.14). This is a pp-wave brane located at $y^4 = \tilde{q}$ and $z^1 = z^2 = z^3 = 0$. The rescaled worldvolume flux is

$$\tilde{F} = \tilde{q} dz^4 \wedge dx^+. \tag{6.18}$$

Different initial choices for the AdS_4 submanifold will lead to branes located at codimension one hypersurfaces in the y^a plane. Since all such branes wrap the same maximal S^2 in the S^5, their Penrose limits will always lead to branes located at $z^1 = z^2 = z^3 = 0$.

6.3 Branes in the plane wave background

Following the general theme of this paper, it is interesting to find the plane wave brane embeddings into the Hpp-wave background directly from the D-brane equations of motion and to derive their preserved supersymmetries explicitly. So let us look for D5-brane embeddings in which the worldvolume coordinates are

$$\xi^i = \left\{x^+, x^-, y^1, y^2, y^3, z^4\right\}, \tag{6.19}$$

and the transverse scalars are

$$X^{\lambda} = \left\{ y^4, z^1, z^2, z^3 \right\}, \tag{6.20}$$

and do not depend on the worldvolume coordinates. We also assume that there is a flux $\tilde{F} = \tilde{q} dz^4 \wedge dx^+$. Then the D-brane equations of motion from (2.21) reduce to

$$\begin{aligned} \partial_i(\sqrt{-M}\theta^{ij}) &= 0; \\ 0 = -\partial_i(\sqrt{-M}G^{ij}\partial_j X^n g_{mn}) &+ \tfrac{1}{2}(\sqrt{-M}G^{ij}\partial_i X^n \partial_j X^p g_{np,m}). \end{aligned} \tag{6.21}$$

since there are no WZ source terms. Substituting in the ansatz, we find that these equations impose no further constraints: to check this note that $\sqrt{-M} = 2$, $G^{++} = 0$ and the only non-zero component of θ^{ij} is $\theta^{-z^4} = \tilde{q}/2$. The constant values of the transverse scalars are left arbitrary (effectively because $G^{++} = 0$).

However, the Penrose limit of the supersymmetric $AdS_4 \times S^2$ branes gives D5-branes located at $(y^4)^2 = \tilde{q}^2$ and $z^1 = z^2 = z^3 = 0$. A natural question to ask is thus whether the plane wave brane is supersymmetric for arbitrary values of the transverse scalars or just for these values. To check this we will use the kappa symmetry projection.

Let us first construct the target space Killing spinors, following closely the analysis given in [9]. Choose the vielbein to be

$$e^{\hat{-}} = 2dx^- + \tfrac{1}{2}((y^a)^2 + (z^a)^2)dx^+; \;\; e^{\hat{+}} = -dx^+; \;\; e^{\hat{a}} = dy^a; \;\; e^{(\widehat{a+4})} = dz^a, \tag{6.22}$$

where we denote tangent space indices by hats. This implies the choice for the tangent space metric component $\eta_{\hat{-}\hat{+}} = 1$. The target space Dirac matrices Γ can then be expressed as

$$\Gamma^{-} = \tfrac{1}{2}\gamma_{+} + \tfrac{1}{4}((y^a)^2 + (z^a)^2)\gamma_{-}; \;\; \Gamma^{+} = -\gamma_{-}; \;\; \Gamma^{y^a} = \gamma_a; \;\; \Gamma^{z^a} = \gamma_{(a+4)}. \tag{6.23}$$

Using the gravitino supersymmetry transformations we find that the target space Killing spinors satisfy $\partial_{-}\epsilon = 0$ and

$$\begin{aligned} (\partial_{y^a} + \tfrac{1}{2}i\gamma_{-a1234})\epsilon = 0; \;\; (\partial_{z^a} + \tfrac{1}{2}i\gamma_{-(a+4)5678})\epsilon &= 0; \\ (\partial_{+} + \tfrac{1}{2}\sum_{a=1}^{4}(\gamma_{-a}y^a + \gamma_{-(a+4)}z^a) + \tfrac{1}{2}i(\gamma_{1234} + \gamma_{5678}))\epsilon &= 0. \end{aligned} \tag{6.24}$$

To derive these equations we have used the fact that ϵ is a negative chirality spinor, so that

$$\gamma_{+-12345678}\epsilon = -\epsilon. \tag{6.25}$$

The solution to these equations is

$$
\begin{aligned}
\epsilon &= (1-\tfrac{1}{2}i\sum_{a=1}^{4}\gamma_-(y^a\gamma_a\gamma_{1234}+z^a\gamma_{(a+4)}\gamma_{5678}))(\cos(\tfrac{1}{2}x^+)-i\sin(\tfrac{1}{2}x^+)\gamma_{1234}) \\
&\quad\times(\cos(\tfrac{1}{2}x^+)-i\sin(\tfrac{1}{2}x^+)\gamma_{5678})(\lambda+i\eta),
\end{aligned} \tag{6.26}
$$

where λ and η are constant real negative chirality spinors.

The kappa symmetry projection is

$$
\epsilon = i\,(\gamma_{+-1238}\epsilon^* - \tilde{q}\gamma_{-123}\epsilon) \tag{6.27}
$$

to be evaluated on the embedding hypersurface which has constant (y^4, z^1, z^2, z^3). The condition must hold at all values of x^+ (since this is a worldvolume coordinate) and so using the explicit form of the Killing spinors (6.26) leads to the conditions

$$
\begin{aligned}
(1+iR)(1+iP)(\lambda+i\eta) &= iQ(1-iP)(\lambda-i\eta); \\
(1+iR)(1+iP)(\gamma_{1234}+\gamma_{5678})(\lambda+i\eta) &= -iQ(1-iP)(\gamma_{1234}+\gamma_{5678})(\lambda-i\eta); \\
(1+iR)(1+iP)\gamma_{12345678}(\lambda+i\eta) &= iQ(1-iP)\gamma_{12345678}(\lambda-i\eta),
\end{aligned} \tag{6.28}
$$

where

$$
\begin{aligned}
Q &= \gamma_{+-1238}; \\
P &= -\tfrac{1}{2}\sum_{a=1}^{4}\gamma_-(y^a\gamma_a\gamma_{1234}+z^a\gamma_{(a+4)}\gamma_{5678}); \\
R &= \tilde{q}\gamma_{-123}.
\end{aligned} \tag{6.29}
$$

The real and imaginary parts of the first and third conditions in (6.28) imply

$$
\begin{aligned}
(QP-1)\lambda+(P+Q+R)\eta &= 0; \\
(P+R-Q)\lambda+(1+QP)\eta &= 0; \\
(1+QP)\lambda+(P+R-Q)\eta &= 0; \\
(P+Q+R)\lambda+(QP-1)\eta &= 0,
\end{aligned} \tag{6.30}
$$

where we have used $P^2 = 0 = RP$ (since $\gamma_-^2 = 0$). These equations then imply the conditions

$$
\lambda = Q\eta; \quad \{P,Q\}\eta = -QR\eta; \quad [Q,R]\eta = 0. \tag{6.31}
$$

The first of these conditions breaks the supersymmetry by one half. The third condition is satisfied automatically since Q and R commute. Explicitly evaluating the anticommutator, the second condition requires that

$$
(\gamma_{-8}y^4 - \gamma_{-12367}z^1 + \gamma_{-12357}z^2 - \gamma_{-12356}z^3)\eta = -\tilde{q}\gamma_{-8}\eta. \tag{6.32}
$$

This condition can be satisfied in two ways. One possibility is $y^4 = -\tilde{q}$ with $z^1 = z^2 = z^3 = 0$, in which case no condition needs to be imposed on η. We note here that had we chosen the negative sign in the kappa symmetry projection (cf equation (4.5)), we would still have gotten a supersymmetric configuration, but located at $y^4 = \tilde{q}$ and $z^1 = z^2 = z^3 = 0$. One should contrast this to the case of the AdS embeddings where only the positive sign in the kappa symmetry projection yielded a supersymmetric solution.

We still need to check that the second condition in (6.28) is satisfied. Using the negative chirality of η and λ we can rewrite the condition as

$$(1 + iR)(1 + iP)\gamma_{1234}\gamma_-\gamma_+(\lambda + i\eta) = -iQ(1 - iP)\gamma_{1234}\gamma_-\gamma_+(\lambda - i\eta). \tag{6.33}$$

Again using $R\gamma_- = P\gamma_- = 0$ this reduces to

$$\gamma_{1234}\gamma_-\gamma_+(\lambda + i\eta) = -iQ\gamma_{1234}\gamma_-\gamma_+(\lambda - i\eta) = i\gamma_{1234}\gamma_-\gamma_+ Q(\lambda - i\eta), \tag{6.34}$$

which is manifestly satisfied when $\lambda = Q\eta$ as in (6.31). The brane embedding then breaks the background supersymmetry by one half, the projection on the spinors being

$$\lambda = \gamma_{+-1238}\eta. \tag{6.35}$$

This is what happens for the Penrose limits of $AdS_4 \times S^2$ branes.

The second possibility to satisfy (6.32) is that we impose the constraint on η

$$\gamma_-\eta = 0. \tag{6.36}$$

The constraint can then be satisfied for arbitrary $(\tilde{q}, z^1, z^2, z^3, y^4)$. The second condition in (6.28) is then automatically satisfied and the embeddings then break the background supersymmetry to one quarter.

7 Other $AdS_{m+1} \times S^{n+1}$ branes and their Penrose limits

Before going on to discuss more generally branes in pp-wave backgrounds we would like to consider other Dp-brane embeddings in $AdS_5 \times S^5$ and their Penrose limits. As we pointed out in section two, provided that there are no WZ source terms, the Dp-brane equations of motion reduce to the constraint that the trace of the second fundamental form of the embedding is zero. Many such embeddings will exist but we want to focus on embeddings of the form $AdS_{m+1} \times S^{n+1}$, which originate from intersections of D$(m+n+1)$-branes with the background D3-branes. All such embeddings are totally geodesic provided the sphere is maximal (the second fundamental form vanishes) and hence they satisfy the equations of motion. The most economic way to explicitly verify this is to work in Poincaré coordinates and choose an ansatz

$$\begin{aligned} \xi^i &= \{x^0, .., x^m, u, \theta_{5-n}, .., \theta_5\}; \\ X^\lambda &= \{x^{m+1}, .., x^3, \theta_1, ..., \theta_{4-n}\}. \end{aligned} \tag{7.1}$$

The equations of motion are then satisfied provided that the transverse scalars in the AdS_5 are constant and the wrapped sphere is maximal. Note that the $AdS_2 \times S^4$ embedding was already found in [56], and can be generalised in an obvious way by putting electric flux on the AdS_2.

The dual interpretation of these branes has not been discussed beyond the $AdS_4 \times S^2$ branes considered in detail here but they should all be understood in terms of higher codimension defects in the field theory. One could derive the effective field theory from appropriate intersections of the (flat space) D$(m+n+1)$-brane and D3-brane worldvolume theories. All these defects should preserve a subgroup of the conformal invariance of the bulk field theory because of the conformal invariance of the induced worldvolume metrics. We discuss the dual dCFTs further in §9.

Consider next the supersymmetry of these embeddings. The kappa symmetry projectors are

$$\Gamma_{(m+1),(n+1)} = \gamma^{0..m4(9-n)..9} K^{\frac{m+n+2}{2}} I \tag{7.2}$$

where we recall that K acts by complex conjugation, I by a multiplication by $-i$. (We could of course just write $K = *$ but sticking to this notation makes the relation with other spinor conventions more manifest.) One follows similar analysis to that given for $AdS_4 \times S^2$ branes to demonstrate that the these branes are one half supersymmetric for $p = 1, 5$ when both m and n are odd, whilst they are supersymmetric for $p = 3, 7$ when both m and n are even. This gives rise to the possibilities listed in Table 1, namely AdS_2, $AdS_3 \times S^1$, $AdS_4 \times S^2$, $AdS_2 \times S^4$, $AdS_5 \times S^3$ and $AdS_3 \times S^5$. The key point of the analysis is that preservation of supersymmetry requires that

$$[\mathcal{P}_\pm, \Gamma_{(m+1)(n+1)}] = 0, \tag{7.3}$$

where $\mathcal{P}_\pm$ are the projections introduced in (4.8). One can easily show that this condition is satisfied by $\Gamma_{(m+1),(n+1)}$ only in the cases we list above. As we discuss in the introduction, the supersymmetric $AdS_{m+1} \times S^{n+1}$ embeddings are in one to one correspondence with the near horizon limits of supersymmetric intersections of D3-branes with other Dp-branes, as one would expect.

Now let us take the Penrose limits of these brane embeddings. We could do this explicitly by writing the embeddings in terms of global coordinates for the background and then applying the appropriate Penrose limit. However, from the $AdS_4 \times S^2$ case we can already see the pattern. Provided that the brane wraps the boosted circle, the $AdS_{m+1} \times S^{n+1}$ brane will be mapped to a pp-wave D(m+n+1)-brane with induced metric

$$ds^2 = -4dx^+dx^- - \left(\sum_{a=1}^{m}(y^a)^2 + \sum_{a=1}^{n}(z^a)^2\right)(dx^+)^2 + \sum_{a=1}^{m}(dy^a)^2 + \sum_{a=1}^{n}(dz^a)^2, \tag{7.4}$$

where the transverse positions are all zero. As we will see below branes along the light cone always preserve at least one quarter of the supersymmetry, even when located at arbitrary transverse positions. They preserve one half of the supersymmetry only in the special cases corresponding to the one half supersymmetric branes in $AdS_5 \times S^5$ listed above, namely when the branes are located at the origin and either $p = 1, 5$ with (m, n) both odd or $p = 3, 7$ with (m, n) both even. The same results were obtained recently in [25] by analyzing open strings in the pp-wave background. Note that the AdS_2 brane cannot wrap the boosted circle and cannot be mapped to a lightcone brane. We will discuss later the Penrose limits of branes which are orthogonal to the boosted circle.

It is interesting to note that Dp-branes which are not supersymmetric as embeddings in $AdS_5 \times S^5$ are mapped to one quarter supersymmetric Dp-branes in the pp-wave background. This enhancement of supersymmetry is rather analogous to the supersymmetry enhancement of $AdS_5 \times T^{p,q}$ found in [12], [13], [14] (and other non-maximally supersymmetric and non supersymmetric backgrounds) in the Penrose limit.

This gives just one group of non-supersymmetric brane embeddings which become supersymmetric in the Penrose limit; we expect there to be many more. For example, in the pp-wave background branes located away from the origin in the transverse coordinates to the lightcone still preserve one quarter of the supersymmetry. It is likely that many of these arise from the Penrose limit of non-supersymmetric branes in the $AdS_5 \times S^5$ background (wrapping non-maximal spheres, say). Understanding this enhancement of supersymmetry in the open string sector would be very interesting.

8 Branes in the pp-wave background

Now let us discuss more generally Dp-brane embeddings in the pp-wave background and their supersymmetry. In this section we find all possible brane embeddings in which the transverse scalars are constants and there is zero worldvolume flux. It is convenient to discuss separately branes with two, one and zero directions along the light cone.

8.1 Light cone branes: $(+,-,m,n)$ branes

First let us consider Dp-brane embeddings whose longitudinal directions include the light cone, whose transverse positions are (arbitrary) constants and which carry no worldvolume flux. Following the discussion around (6.21) one can show that (almost) any Dp-brane embedding, with arbitrary p and constant transverse scalars will satisfy the D-brane equations of motion.

Suppose the Dp-brane longitudinal to the light cone has m longitudinal directions amongst the y^a, labelled by $(a_1..a_m)$ and n longitudinal directions amongst the z^a, labelled by $(b_1...b_n)$; for convenience of notation we will call this an $(+,-m,n)$ Dp-brane. Then the allowed constant embeddings can be summarised as

$$\begin{array}{lcl} D1 & : & (+,-,0,0) \\ D3 & : & (+,-,0,2) \;\; (+,-,1,1) \;\; (+,-,2,0) \\ D5 & : & (+,-,1,3) \;\; (+,-,2,2) \;\; (+,-,3,1) \\ D7 & : & (+,-,2,4) \;\; (+,-,3,3) \;\; (+,-,4,2) \\ D9 & : & (+,-,4,4) \end{array} \tag{8.1}$$

where in each case the transverse positions are arbitrary. In each case the induced worldvolume metric is a pp-wave, as for the $(+,-,3,1)$ brane discussed previously. Note that the D9-brane fills the entire spacetime.

The only possibilities not allowed in this table are $(+,-,4,0)$ and $(+,-,0,4)$ D5-branes. This is because in these cases there is a non-zero WZ current which acts as a source for the gauge field on the worldvolume, and so it is not consistent to set this gauge field to zero. We will return to these exceptional cases below: they are directly related to the baryon vertex [57] in the dual theory.

All of the branes in the table above with $m > 0$ originate as $AdS_{m+1} \times S^{n+1}$ branes. The exceptional cases of $(+,-,0,0)$ and $(+,-,0,2)$ cannot originate from AdS embeddings; instead these branes extend along the time direction in AdS and a maximal sphere in the

S^5. The explicit forms of these embeddings are most easily found in global coordinates: the branes extend along the τ direction at $\rho = \psi = \chi = 0$.

Now consider the supersymmetry of these embeddings. The kappa symmetry projector is

$$\Gamma = \gamma_{+-a_1..a_m b_1..b_n} K^{\frac{p+1}{2}} I \equiv Q K^{\frac{p+1}{2}} I. \tag{8.2}$$

Recall that K acts by complex conjugation, I by multiplication by $-i$ and note that $Q^2 = (-1)^{\frac{p-1}{2}}$. Following a similar analysis to that around (6.28), we find that necessary and sufficient conditions for the kappa symmetry projection to hold are

$$\eta = Q\lambda; \qquad (QP + (-)^{\frac{p-1}{2}} PQ)\eta = 0; \qquad (-)^{\delta} = (-)^{\frac{p+1}{2}}, \tag{8.3}$$

where the projections must hold on the worldvolume and P is defined in (6.29). δ is one if (m, n) are both odd and is zero if (m, n) are both even. The first condition breaks the supersymmetry by one half by relating real and imaginary parts of the constant spinors. The second condition will not impose further constraints provided that Q and P anticommute on the worldvolume for $p = 1, 5$ and commute on the worldvolume for $p = 3, 7$. If these (anti)commutation conditions are not satisfied we can still satisfy the second condition by imposing the further projection $\gamma_{-}\eta = 0$, which breaks the supersymmetry down to one quarter.

Explicitly evaluating the (anti)commutators for each p and (m, n) split of the indices we find the following results. The following branes preserve one half of the supersymmetry when they are located at the origin in the transverse coordinates:

$$\begin{array}{lcll} D3 & : & (+,-,0,2) & (+,-,2,0) \\ D5 & : & (+,-,1,3) & (+,-,3,1) \\ D7 & : & (+,-,2,4) & (+,-,4,2) \end{array} \tag{8.4}$$

All other possibilities given in the previous table preserve only one quarter supersymmetry. This agrees with the analysis of open strings in the pp-wave background reported in [25].

Just as for the $(+,-,3,1)$ branes discussed previously, it is likely that switching on specific constant worldvolume fluxes may allow us to move the branes away from the origin in the transverse directions whilst preserving one half supersymmetry. It is certainly true that, as in the previous analysis, one can still satisfy the field equations with constant transverse scalars if one switches on constant fluxes F_{+a} on the worldvolume. We have not explored in detail under what conditions such embeddings with constant fluxes are supersymmetric.

Before leaving the lightcone branes, let us briefly discuss the exceptional case of $(+,-,0,4)$ branes. Recall that the pulled back RR flux acts as a source for flux on the worldvolume. One can verify that the field equations are satisfied for (arbitrary) constant transverse scalars provided that one switches on a worldvolume flux such that

$$F = f_a dx^+ \wedge dy^a \qquad \sum_{a=1}^{4} \partial_a f_a = 8. \tag{8.5}$$

So although a "constant" $(+,-,0,4)$ embedding does not exist there is still a simple $(+,-,0,4)$ embedding with worldvolume flux. The next question is whether this embedding preserves supersymmetry: the analysis follows closely that given for $(+,-,3,1)$ branes with flux and we find the supersymmetry conditions reduce to those given in (6.31), namely

$$\lambda = Q\eta; \quad \{P, Q\}\eta = -QR\eta; \quad [Q, R]\eta = 0, \tag{8.6}$$

where P is as in (5.30) but now

$$Q = \gamma_{+-1234}; \quad R = \epsilon^{abcd} f_a \gamma_{-bcd}. \tag{8.7}$$

Now terms in P involving the worldvolume coordinates y^a actually commute with Q, and hence the second condition in (8.6) can only be satisfied everywhere on the worldvolume if we project $\gamma_- \eta = 0$. The third condition in (8.6) is automatically satisfied since the commutator vanishes, and hence these embeddings preserve one quarter of the supersymmetry, regardless of the transverse positions. Thus it is likely that the corresponding embeddings in AdS (D5-branes wrapping the S^5 with flux on the sphere or D5-branes wrapping $AdS_5 \times S^1$ with flux on the AdS_5) are not supersymmetric.

In fact one knows that there must be one half supersymmetric branes whose longitudinal directions are (x^+, x^-, z^a) for which the transverse scalar $r = \sqrt{y^a y^a}$ is *not* constant. These will arise from the Penrose limits of the branes found in [58] which correspond to the baryon vertex in the dual SYM theory [57]. We leave these for future investigation.

8.2 Instantonic branes: (m, n) branes

Now let us consider so-called instantonic branes which are transverse to the light cone directions. Such branes would arise from the Penrose limit of Euclidean branes in AdS. Let us again take the transverse scalars to be constant and assume there is no worldvolume flux. Solving the D-brane equations of motion, and employing the notation (m, n) we find the following possibilities

$$\begin{array}{llll} D1 & : & (0,2)\ (1,1)\ (2,0) \\ D3 & : & (1,3)\ (2,2)\ (3,1) \\ D5 & : & (2,4)\ (3,3)\ (4,2) \\ D7 & : & (4,4). \end{array} \tag{8.8}$$

In each case the induced worldvolume metric is just the flat Euclidean metric. We must leave out the $(4, 0)$ and $(0, 4)$ D3-branes for reasons akin above: the background RR flux acts as a source for worldvolume scalars.

Now consider the supersymmetry of these embeddings. We run into a problem when trying to use the kappa symmetry projection: the Euclideanised kappa symmetry projector has not (as far as we know) been constructed in the literature. To construct such a projector one may Wick rotate the worldvolume theory using the results in [59, 60, 61] and then demand that the action is invariant under the new Euclideanised kappa symmetry. A similar approach was taken in [62] when studying the supersymmetry of D-instantons.

Such instantonic branes were constructed as closed string boundary states [19] and it was found that the following branes preserve half the supersymmetry when located at the origin in the transverse spatial coordinates and at arbitrary positions on the lightcone

$$D1 : (0,2), (2,0) \ \ D3 : (1,3), (3,1) \ \ D5 : (2,4), (4,2). \tag{8.9}$$

All other possibilities preserve one quarter supersymmetry. It would be interesting to investigate corresponding (Euclidean) brane embeddings in $AdS_5 \times S^5$ which are as yet unknown. These correspond to near horizon limits of Euclidean D-branes intersecting with the background D3-branes and are likely to have topologies of the form $H^m \times S^n$.

8.3 Branes along one lightcone direction: $(+_\gamma, m, n)$ branes

Although in string theory in light cone gauge only branes totally transverse or totally longitudinal to the light cone are accessible, branes along only one light cone direction could also exist. Let us take the brane to lie along $x^- = \gamma x^+$. Such branes are certainly physically interesting objects, arising from the Penrose limit of branes in $AdS_5 \times S^5$ which are rotating in the circle direction, at a speed implied by γ. Branes which rotate at the speed of light in the direction of the boost have $\gamma = 0$ and lie along the x^+ direction at some constant value of x^-. These could be called $(+, m, n)$ branes, following the previous notation.

Note that branes for which $\gamma \to \infty$, in other words $(-, m, n)$ branes for which x^+ is strictly constant, and for which the other transverse scalars are constants would have degenerate worldvolume metrics, since $g_{--} = 0$, and hence are not admissible solutions. All other possibilities in the range $0 \leq \gamma < \infty$ could in principle be realized and would correspond to branes rotating at speeds depending on γ along the boosted circle.

To find such branes from our field equations, let us take an ansatz in which the brane extends along (x^+, m, n) with the lightcone transverse scalar being $x^- = x_0^- + \gamma x^+$ and all the other transverse scalars constant. The D-brane field equations permit solutions *only* when the transverse scalars are zero and in the following cases

$$\begin{array}{lcl} D1 & : & (+_\gamma, 0, 1) \ \ (+_\gamma, 1, 0) \\ D3 & : & (+_\gamma, 1, 2) \ \ (+_\gamma, 2, 1) \\ D5 & : & (+_\gamma, 2, 3) \ \ (+_\gamma, 3, 2) \\ D7 & : & (+_\gamma, 3, 4) \ \ (+_\gamma, 4, 3), \end{array} \tag{8.10}$$

where we use as shorthand $+_\gamma$ to indicate the lightcone direction wrapped by the brane. The value of γ is left arbitrary by the field equations though as mentioned above the solution with x^+ strictly constant is not admissible because the induced metric degenerates. The value of x_0^- is also an an arbitrary integration constant. These branes all have induced metrics:

$$ds^2 = -(r^2 + 4\gamma)(dx^+)^2 + dr \cdot dr, \tag{8.11}$$

where $r^2 = \sum_{a=1}^m (y^a)^2 + \sum_{a=1}^n (z^a)^2$. As in the previous discussions, several possibilities are missing from this table, because of the coupling to the RR flux. We will consider below $(+_\gamma, 0, 3)$ and $(+_\gamma, 3, 0)$ branes, for which the RR flux couples to a transverse scalar. $(+_\gamma, 1, 4)$ and $(+_\gamma, 4, 1)$ branes are also missing from this table, because the RR flux again acts as a source for the worldvolume flux, and we do not consider them here.

Starting from our $AdS_{m+1} \times S^{n+1}$ embeddings, if the boost circle is not contained in the wrapped sphere, we find that the Penrose limit of the brane gives the $(+_\gamma, m, (n+1))$ branes found above but with $\gamma \to \infty$.

The simplest case to consider explicitly is the AdS_2 brane. This can be embedded in global coordinates by taking the brane to extend along τ and ρ at $\chi = \psi = 0$, $\phi = const$ and at constant position in the S^5. In particular this means that θ_5 is constant and hence when we change to the lightcone coordinates

$$dx^+ = \frac{dx^-}{R^2} \tag{8.12}$$

on the brane. Now we take the limit $R \gg 1$, giving an induced metric on the brane

$$ds_2^2 = -(r^2 + 4R^2)(dx^+)^2 + dr^2, \tag{8.13}$$

with $x^- = x_0^- + R^2 x^+$. This indeed reproduces the metric for the $(+_\gamma, 1, 0)$ brane found above, though in the Penrose limit we would need to take $R \to \infty$. In this limit the brane lies strictly parallel to the x^- lightcone direction; then the induced metric must be degenerate (since $dx^+ = 0$) and furthermore the brane has infinite energy as measured by the lightcone Hamiltonian. Note that the infinite boost means that branes which are static with respect to the boost will always be mapped to branes along x^- in the pp-wave background.

Now let us consider the supersymmetry of these embeddings. Whenever we take the Penrose limit of a (static) $AdS_{m+1} \times S^{n+1}$ branes using a circle transverse to the brane, the resulting brane will have a degenerate metric and is not an admissible solution. Even branes which were originally half supersymmetric will not appear as admissible embeddings in the pp-wave background.

Branes in the pp-wave background with a finite value of γ originate from branes which were rotating close to the speed of light about the boosted circle. Generically rotating or boosted branes are not supersymmetric and we will find this is true here: virtually none of the $(+_\gamma, m, n)$ branes preserve any supersymmetry even when located at the origin in transverse coordinates. The only exceptions are the $(+, 0, 1)$ and $(+, 1, 0)$ branes as well as the (exceptional) $(+, 3, 0)$ branes discussed in the next section.

The kappa symmetry projector for the $(+_\gamma, 1, 0)$ branes is

$$\Gamma = i \frac{1}{(r^2 + 4\gamma)^{\frac{1}{2}}} (\gamma_+ - \tfrac{1}{2}(r^2 + 4\gamma)\gamma_-)\gamma_1 K \equiv i(Q_+ + Q_-)K, \tag{8.14}$$

where in the latter expression we introduce $Q_\pm$ which depend on $\gamma_\pm$ respectively. Let us write the projection as

$$\Gamma(1 + iP)\psi(x^+) = (1 + iP)\psi(x^+), \tag{8.15}$$

where P is given in (5.30) and

$$\psi(x^+) = (\cos(\tfrac{1}{2}x^+) - i\sin(\tfrac{1}{2}x^+)\gamma_{1234})(\cos(\tfrac{1}{2}x^+) - i\sin\tfrac{1}{2}x^+\gamma_{5678})(\lambda + i\eta) \tag{8.16}$$

is a spinor of negative chirality. Using the nilpotence of γ_- this reduces to

$$i(Q_+ + Q_- - iQ_+ P)\psi^*(x^+) = (1 + iP)\psi(x^+). \tag{8.17}$$

Now this condition must hold everywhere on the worldvolume, namely for all r. Writing P on the worldvolume as

$$P = -\tfrac{1}{2} r \gamma_{-234} \tag{8.18}$$

(all other terms vanish when the transverse scalars are zero) we find that necessary and sufficient conditions for (8.17) to hold are that $\gamma = 0$ and in addition

$$\gamma_+\psi(x^+) = 0; \qquad \gamma_{1234}\psi(x^+)^* = \psi(x^+), \tag{8.19}$$

which can be satisfied by

$$\gamma_{1234}\lambda = \lambda; \qquad \gamma_{1234}\eta = -\eta; \qquad \gamma_+\lambda = \gamma_+\eta = 0. \tag{8.20}$$

The projections break the supersymmetry to one quarter maximal.

Now let us consider the origins of these 1/4 supersymmetric branes in $AdS_5 \times S^5$. The $(+,1,0)$ branes come from the following embedding: use global coordinates for AdS_5 and extend the string along (ρ,τ) with the other AdS_5 coordinates χ, ψ and ϕ fixed. For the transverse positions in the S^5, take all $\theta_a = \frac{1}{2}\pi$, except $\theta_5 = \tau$. The induced worldvolume metric is

$$ds^2 = R^2(-\sinh^2\rho d\tau^2 + d\rho^2), \tag{8.21}$$

which is an AdS_2 metric. Since the brane rotates at the speed of light around the θ_5 circle, we will refer to this brane as a rotating AdS_2 string.

To find the origin of the $(+,0,1)$ brane it is convenient to first introduce the following coordinates on the S^5:

$$ds^2 = (d\theta^2 + \sin^2\theta d\Omega_3^2 + \cos^2\theta d\theta_5^2). \tag{8.22}$$

Then put the D1-brane at $\rho = 0$ in the AdS_5 along τ and along θ in the S^5 at fixed position on the S^3 with $\theta_5 = \tau$. The induced metric in this case is just

$$ds^2 = R^2(-\sin^2\theta d\tau^2 + d\theta^2), \tag{8.23}$$

which we will refer to as a rotating dS_2 string (the induced metric is de Sitter).

Note that there is no reason to suppose that these embeddings should be supersymmetric *a priori* since they do not come from any standard intersecting brane system. Since we find the corresponding pp-wave embedding is only one quarter supersymmetric these embeddings in AdS preserve at most one quarter supersymmetry and possibly none. It would be interesting to check their supersymmetry explicitly.

These rotating D-strings are expected to correspond to magnetic monopoles from the gauge theory point of view. Since the branes are rotating about a circle, the states also carry charge with respect to the corresponding $SO(2)$ R symmetry. Note that the static AdS_2 brane considered above also has a mass of the same order but it does not carry any charge with respect to the R symmetry; it must therefore correspond to a monopole with no R charge, which is why its lightcone mass diverges in the Penrose limit. Although the masses of the rotating D-strings also diverge in the Penrose limit ($\lambda \to \infty$ with g_{YM}^2 fixed), they can still have finite lightcone Hamiltonian because the R-charge can cancel with the mass term.

8.4 An exceptional case: the $(+, 0, 3)$ branes and giant gravitons in AdS

Finally let us consider the $(+, 3, 0)$ and $(+, 0, 3)$ branes which are missing from the above classification, because of their coupling of the RR flux to the worldvolume scalars. It turns out that there is a simple D3-brane embedding along one direction of the lightcone. Let us treat the $(+, 3, 0)$ brane; as usual the other case follows by simply exchanging of the y^a and z^a coordinates.

Take the longitudinal directions to be (x^+, Ω_3), where we now use polar coordinates (r, Ω_3) to describe the R^4 parametrised by y^a. Then the field equations are satisfied provided that $y^a y^a = r^2$ is constant (the brane wraps an S^3 in this R^4) and the transverse scalars z^a are zero. We will also take the lightcone transverse scalar x^- to be constant, so that the brane rotates at the speed of light along the boosted circle.

The induced worldvolume metric on the brane is just

$$ds^2 = -r^2(dx^+)^2 + r^2 d\Omega_3^2, \tag{8.24}$$

which is an Einstein universe of radius r. Thus the effect of the coupling to the flux is that the spatial sections of the $(+, 3, 0)$ brane are spherical rather than flat. Note that r is not necessarily zero: surprisingly the field equations allow the brane to have a finite radius, although one would naively expect it to shrink. This stabilisation results from the coupling to the background RR flux.

To check supersymmetry, note first that the projection operator is

$$\Gamma = ir^{-1}(\gamma_+ - \tfrac{1}{2}r^2\gamma_-)\gamma_{234} = iQ, \tag{8.25}$$

where $Q^2 = -1$. Then note that on the worldvolume $z^a = 0$ with r constant we can rewrite P in (5.30) as

$$P = -\tfrac{1}{2}r\gamma_-\gamma_{234} \tag{8.26}$$

where we relate gamma matrices in polar and Cartesian frames by $\gamma_r = \gamma_1$, $\gamma_{\theta_1} = r\gamma_2$ and so on. Now the kappa symmetry projection is

$$iQ(1 + iP)\psi(x^+) = (1 + iP)\psi(x^+), \tag{8.27}$$

where we absorb the x^+ dependence of the Killing spinor into a spinor $\psi(x^+)$. Using the nilpotence of $\gamma-$ this reduces to

$$i\tilde{Q}(1+iP)\psi(x^+) = \psi(x^+) \tag{8.28}$$

where $\tilde{Q} = r^{-1}\gamma_{+234}$. This can be solved for any r at all values of the worldvolume coordinate x^+ by taking

$$\gamma_+\psi(x^+) = 0. \tag{8.29}$$

This projects out half of the λ and η spinors contained in $\psi(x^+)$ and kills the first term in (8.28). Noting that

$$-\tilde{Q}P\psi(x^+) = \tfrac{1}{2}\gamma_+\gamma_-\psi(x^+) = \psi(x^+), \tag{8.30}$$

where we use (8.29) and $\{\gamma_+, \gamma_-\} = 2$, we see that (8.28) is indeed satisfied. Thus, perhaps somewhat surprisingly, the brane embeddings preserve one half supersymmetry for any value of r. In a slight abuse of notation, these could be called $(+,3,0)$ branes (the abuse is that the branes extend along an S^3 and not an R^3 of the R^4). Embeddings closely related to these in which r is a function of x^+ will be the missing $(4,0)$ instantonic branes, though the embeddings are no longer constant: indeed they must be explicitly time dependent.

Now we have argued that branes which preserve one half supersymmetry will originate from supersymmetric configurations in $AdS_5 \times S^5$. So this analysis indicates that there is are stable rotating D3-branes in $AdS_5 \times S^5$. These branes correspond to giant gravitons [35, 36, 37]. Again the explicit form of the embedding is easiest to find using global coordinates. The $(+,3,0)$ embeddings originate from branes which extend along the directions (τ, χ, ψ, ϕ) in the AdS_5 at *arbitrary* radius ρ_0, with $\theta_5 = \tau$ parametrising the rotation in the S^5 and all other angular coordinates in the S^5 being $\pi/2$. Then the induced worldvolume metric is just

$$ds^2 = R^2 \sinh^2\rho_0[-(d\tau)^2 + (d\chi^2 + \cos^2\chi d\psi^2 + \sin^2\chi d\phi^2)], \tag{8.31}$$

which is an Einstein universe, just as for the Penrose limit of the brane. Note that if the brane was not rotating it would be forced towards $\rho \to 0$; the rotation stabilises it at finite ρ_0. It was shown in [36, 37] that this embedding is half supersymmetric.

To find the origin of the $(+,0,3)$ embedding in $AdS_5 \times S^5$ it is easiest to use global coordinates for the AdS and introduce coordinates on the S^5

$$ds^2 = R^2[d\theta^2 + \sin^2\theta d\Omega_3^2 + \cos^2\theta d\theta_5^2]. \tag{8.32}$$

The following ansatz satisfies the field equations: the brane wraps the S^3 in this S^5 and rotates at the speed of light in the θ_5 direction, so that $\tau = \theta_5$, with all other transverse scalars (the other coordinates on the AdS_5 and θ) constant. Then the field equations are satisfied with $\rho = 0$ but *arbitrary* $\theta = \theta_0$. The induced metric on the brane is then

$$ds^2 = R^2 \sin^2 \theta_0 [-(d\tau)^2 + d\Omega_3^2], \tag{8.33}$$

which is again an Einstein universe. This embedding was also discussed in [36, 37] and shown to be half supersymmetric.

Now this brane is precisely the giant graviton discussed in [35]: it must be, because it is a BPS spherical D3-brane with the right quantum numbers. A simple way of seeing this is to note that the light cone Hamiltonian vanishes:

$$H = i(\partial_\tau - \Omega \partial_{\theta_5}) = (M - J\Omega) = 0 \tag{8.34}$$

where M is the mass, J is the angular momentum and Ω is the angular velocity (equal to one here, since the brane rotates at the speed of light). Reinstating the factors arising from the D-brane action, we see that this implies that

$$M = J \sim \frac{R^4}{g} \sim N, \tag{8.35}$$

which is indeed the behaviour we expect from the giant graviton [35].

Finally, let us say that the whole story of D-brane embeddings in pp-wave and $AdS \times S$ backgrounds seems to be very rich and deserves much more investigation. We hope to have pointed out a number of avenues to pursue: we have classified all constant brane embeddings in the pp-wave background but embeddings arising from the baryon vertex branes, those with (non)constant fluxes on the worldvolume and rotating branes should be explored much further. By essentially inverting the Penrose limit, these branes in the pp-wave background will lead us to previously unknown brane embeddings in $AdS_5 \times S^5$, beyond the ones discussed here. Such embeddings could in turn lead us to new results in the corresponding dual field theories.

9 D-branes from gauge theory

The pp-wave limit of $AdS_5 \times S^5$ has attracted interest principally because one can directly construct the light-cone string theory in the pp-wave background from gauge theory [11]. The authors of [11] constructed the light-cone closed string states using specific operators of the $\mathcal{N} = 4$ SYM. A natural question is how to construct the D-brane states we found here using gauge theory. Since D-branes capture non-perturbative aspects of string theory this question is of paramount interest. We will present here a construction of all (longitudinal) D-branes appearing in top part of Table 1 of the introduction. As this paper was being typed, two very interesting papers appeared, [26] and [28], where the construction we outline below was carried out in detail for the case of the D7-$(+,-,4,2)$ brane and the D5-$(+,-,3,1)$ brane, respectively.

As already discussed, there is a one to one correspondence between supersymmetric intersections ($Dq \perp D3$), supersymmetric AdS D-branes in $AdS_5 \times S^5$ and Dq-branes along the light cone in the pp-wave background. We will make use of all three to construct the latter Dq-branes from the gauge theory.

Firstly, following the arguments in [3, 4], one expects an AdS/dCFT duality for all cases appearing in table 1. That is, we expect a duality between the bulk theory on $AdS_5 \times S^5$ together with a Dq-brane probe and the boundary theory $\mathcal{N} = 4$ SYM with a defect CFT on the boundary of the AdS-embedding. The case of $(3|D7 \perp D3)$ (leading to the D7-$(+,-,4,2)$ brane in the pp-wave background) is somewhat special in that the "defect CFT" is actually of "co-dimension zero", i.e. the D3-branes lie entirely on the worldvolume of the D7-brane and the fields coming from 3-7 and 7-3 strings are localized on the worldvolume of the D3-brane. As is well known, one needs to include an orientifold plane in this case. The corresponding duality is well understood [63], [64] and we refer to these papers for further details.

In the limit discussed in [4], the bulk theory captures the physics of the closed strings and of the q-q open strings, whilst the boundary theory captures that of the 3-3, 3-q and q-3 open strings. The boundary action can be obtained from the action of Dq-D3 system upon taking the near-horizon limit discussed in [4]. In particular, we note that the 3-q and q-3 open strings will give rise to hypermultiplets in the fundamental of $SU(N)$. These defect fields interact amongst themselves and with the restriction of the $\mathcal{N} = 4$ vector multiplet to the defect. As argued in [4], the defect theory will capture holographically the physics of the q-q open strings.

To make the duality precise one needs to develop a dictionary between bulk fields and boundary operators. In particular, the worldvolume fields of the Dq-brane reduced over the wrapped sphere should match with (certain) operators of the defect theory. This has been done explicitly for the $(2|D5 \perp D3)$ and the $(3|D7 \perp D3)$ cases in [4] (as discussed in §5) and in [64], respectively. Notice that all configurations in the top part of Table 1 have four Neumann-Dirichlet (ND) boundary conditions and the ones in the bottom part have eight. This means that all AdS/dCFT pairs of the top/bottom part of the table are formally connected by T-duality (formally because one would have to wrap the branes on tori to perform the T-duality, an operation which in general does not commute with the near horizon limit). This implies that a consistent field/operator matching is guaranteed.

Having argued the existence of an AdS/dCFT in each case, we now take the pp-wave limit of these configurations. The construction of the closed string spectrum proceeds as in [11] and we shall not repeat it here; in what follows we use the notation of [11] when referring to the closed string sector. We only mention that the closed string vacuum is constructed from the Z's and the oscillators from insertions of $D_i Z$ and ϕ_i, where $Z = X^4 + iX^5, \bar{Z}, \phi^i = X^{i-5}$, and $X^i, i = 4, .., 9$, are the six scalars of $\mathcal{N} = 4$ SYM.

As we have seen, AdS embeddings are mapped to D-branes along the light cone in the pp-wave background, suggesting that the D-brane states can be constructed using defect fields. In particular, to construct the open string states one needs to have fields in the fundamental. For the $ND = 4$ cases these are supplied by the hypermultiplets localized on the defect. To construct the appropriate states we will need to know the J charge of the hypermultiplets with respect to $SO(2)$ associated to the circle along which we boost. The easiest way to compute this is to go back to the original Dq-D3 system and find the J charge by looking at the way the hypermultiplets are constructed from the fermionic zero modes of the q-3 and 3-q strings. The computation is identical in all three cases, i.e. D3, D5 and D7 branes and so it will suffice to consider just one case. Since the D5 and D7 branes have appeared already in the literature let us discuss explicitly here the D3-brane. The intersecting D3-D3' brane configuration is illustrated in the table below. We remind

	0	1	2	3	4	5	6	7	8	9
D3'	N	N	N	N	D	D	D	D	D	D
D3	N	N	D	D	N	N	D	D	D	D

Table 2: D3-brane intersection

our readers that to get to the D3 brane in the pp-wave background from this configuration we will first need to go to the near-horizon limit of the D3' branes, with the 45 directions of the D3 brane extending along the radial direction in AdS_5 and along an S^1 in S^5. This produces the half supersymmetric $AdS_3 \times S^1$ embedding in $AdS_5 \times S^5$, as we have discussed. We then take the pp-wave limit by boosting along this same circle, which leads to the half supersymmetric D3-$(+, -, 2, 0)$ brane of the pp-wave background. Thus the $SO(2)$ that participates in the pp-wave limit acts as a rotation in the 4-5 plane.

Now let us go back to the original intersecting D-brane system to find the R-charges of the defect hypermultiplets. This follows from an analysis exactly analogous to that given in p. 162 of [65], where the 5-9 and 9-5 strings of the D5-D9 system were studied. The massless states of the 3-3' system form a "half-hypermultiplet". The scalars $q_i, i = 1, 2$, are in the fundamental of $SU(N)$ and transform as $(1/2, 1/2)$ and $(-1/2, -1/2)$ under rotations in the 45 and 23 planes. Similarly, the 3'-3 strings yield yield another "half-hypermultiplet" containing two scalars $\bar{q}^i$ in the anti-fundamental of $SU(N)$ and transforming the same way as q_i under rotations in the 45 and 23 planes. Thus we find that q_1 and $\bar{q}^1$ have $J = 1/2$ and other two $J = -1/2$. Both q_i and $\bar{q}^i$ are singlets under the $SO(4)$ that rotates the DD directions. Analogous analysis and results apply to the the D5 and D7 branes.

We now propose that the open string vacuum is all cases is given by

$$|0; p^+\rangle \leftrightarrow \text{tr } \bar{q}^1 Z^J q_1 \tag{9.1}$$

Note that the Z appearing in this formula denotes the restriction of Z to the defect. This formula is schematic in the case of the D7-brane since we do not take into account the orientifold; a more precise formula for this case can be found in [26].

To compute the light-cone energy we need to know the dimension Δ of the q_i and Z. The dimension of Z is always one, and the dimension of q_i depends on the spacetime dimension: $\Delta_d = (d-2)/2$. Hence the light-cone energy of (9.1) will vary from case to case,

$$D3 : (\Delta - J)_0 = -1, \qquad D5 : (\Delta - J)_0 = 0, \qquad D7 : (\Delta - J)_0 = 1. \tag{9.2}$$

One should compare this with the ground state energy of the corresponding D-brane. The number of bosonic massive zero modes is equal to $(p-1)$ for a Dp brane, and each of them contributes $1/2$ to the ground state energy. In all three cases there are four massive fermionic zero modes, each contributing $-1/2$ to the ground state energy. Combining these results we find exact agreement with (9.2)!

The discussion so far refers to the branes originating from $ND = 4$ systems. Let us now discuss the $ND = 8$ systems. In this case the massless spectrum of the q-3 and 3-q strings consists of two fermions, χ and $\bar{\chi}$, one in the fundamental and the other in the anti-fundamental of $SU(N)$[66, 67] . These originate from the R sector and are singlets under rotations in the ND directions; we hence conclude that both χ and $\bar{\chi}$ have J charge equal to zero. We now propose that the open string vacuum is given by

$$|0; p^+\rangle \leftrightarrow \text{tr } \bar{\chi} Z^J \chi \tag{9.3}$$

Notice that the canonical dimension of χ in d spacetime dimensions is $\Delta_\chi = (d-1)/2$ and it hence follows that the light-cone energy of (9.3) in all cases[6] is given exactly by (9.2)!

One may now follow the discussion in [11] and construct oscillators by insertions of ϕ^i and the covariant derivatives of $D_i Z$. In the case of the open string states one would insert the restriction of these fields to the defect. To construct the non-zero modes one needs to add phases to the closed string states, as in [11], and cosines and sines for open strings with Neumann and Dirichlet boundary conditions, respectively, as in [26, 28].

One still has to specify which operator should be inserted to obtain a given oscillator. This follows from the form of the original intersection: one inserts a $D_i Z$ in the direction that was parallel to the i worldvolume direction of the D3-brane in the original intersection, and ϕ^i in the directions that were parallel to the ϕ^i direction transverse to the D3-brane. These rules follow from the decomposition of the vector multiplet of the original D3-brane into other multiplets when restricted to the defect.

These rules apply equally to the $ND = 4$ and $ND = 8$ cases. For illustrative purposes we present the case of the D3-$(+,-,2,0)$ brane. The oscillators of the zero modes in the Neumann directions are

$$\begin{aligned} a_0^{0\dagger}|0;p^+\rangle &\leftrightarrow \frac{1}{\sqrt{J}}\sum_{l=0}^{J+1}\frac{1}{N^{J/2+1}}\bar{q}^1 Z^l (D_0 Z) Z^{J+1-l} q_1 \\ a_0^{1\dagger}|0;p^+\rangle &\leftrightarrow \frac{1}{\sqrt{J}}\sum_{l=0}^{J+1}\frac{1}{N^{J/2+1}}\bar{q}^1 Z^l (D_1 Z) Z^{J+1-l} q_1 \end{aligned} \tag{9.4}$$

where we remind the reader that the D3 brane extends over the $(+,-)$ directions as well (the superscript 0 in $a_0^{0\dagger}$ does not mean that this is a time-like direction), whereas for the Dirichlet directions we have

$$\begin{aligned} a_0^{2\dagger}|0;p^+\rangle &\leftrightarrow \frac{1}{\sqrt{J}}\sum_{l=0}^{J+1}\frac{1}{N^{J/2+1}}\bar{q}^1 Z^l (D_2 Z) Z^{J+1-l} q_1 \\ a_0^{3\dagger}|0;p^+\rangle &\leftrightarrow \frac{1}{\sqrt{J}}\sum_{l=0}^{J+1}\frac{1}{N^{J/2+1}}\bar{q}^1 Z^l (D_3 Z) Z^{J+1-l} q_1 \\ a_0^{i+5\dagger}|0;p^+\rangle &\leftrightarrow \frac{1}{\sqrt{J}}\sum_{l=0}^{J+1}\frac{1}{N^{J/2+1}}\bar{q}^1 Z^l \phi^i Z^{J+1-l} q_1 \end{aligned} \tag{9.5}$$

One can check that the assignments in [26] and [28] follow from this rule.

Finally the non-zero mode excitations are obtained by adding cosines in the Neumann

[6] One may formally include the D3-brane as well by considering the "defect theory" to be a matrix theory.

directions and sines in the Dirichlet directions [26, 28]

$$
\begin{aligned}
a_n^{0\dagger}|0;p^+\rangle &\leftrightarrow \frac{1}{\sqrt{J}}\sum_{l=0}^{J+1}\frac{\sqrt{2}\cos\left(\frac{n\pi l}{J}\right)}{N^{J/2+1}}\bar{q}^1 Z^l (D_0 Z) Z^{J+1-l} q_1 \\
a_n^{1\dagger}|0;p^+\rangle &\leftrightarrow \frac{1}{\sqrt{J}}\sum_{l=0}^{J+1}\frac{\sqrt{2}\cos\left(\frac{n\pi l}{J}\right)}{N^{J/2+1}}\bar{q}^1 Z^l (D_1 Z) Z^{J+1-l} q_1 \\
a_n^{2\dagger}|0;p^+\rangle &\leftrightarrow \frac{1}{\sqrt{J}}\sum_{l=0}^{J+1}\frac{\sqrt{2}\sin\left(\frac{n\pi l}{J}\right)}{N^{J/2+1}}\bar{q}^1 Z^l (D_2 Z) Z^{J+1-l} q_1 \\
a_n^{3\dagger}|0;p^+\rangle &\leftrightarrow \frac{1}{\sqrt{J}}\sum_{l=0}^{J+1}\frac{\sqrt{2}\sin\left(\frac{n\pi l}{J}\right)}{N^{J/2+1}}\bar{q}^1 Z^l (D_3 Z) Z^{J+1-l} q_1 \\
a_n^{i+5\dagger}|0;p^+\rangle &\leftrightarrow \frac{1}{\sqrt{J}}\sum_{l=0}^{J+1}\frac{\sqrt{2}\sin\left(\frac{n\pi l}{J}\right)}{N^{J/2+1}}\bar{q}^1 Z^l \phi^i Z^{J+1-l} q_1
\end{aligned}
\tag{9.6}
$$

One may proceed to compute the anomalous dimensions of these operators as in [11, 26, 28]. For this one would need to know the precise form of the interactions between the boundary and defect fields. One may use T-duality to obtain these interactions from the interactions given in [4]. This suggests that the arguments in [28] carry over for this case as well. Another avenue is to relate this system to the D1-D5 system. We leave a detailed investigation for future work.

We have presented the construction of all half supersymmetric D-branes that are visible in the light-cone gauge. It would be very interesting to understand how to construct the rotating D1 and D3 branes. We expect these to appear as non-perturbative states.

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A Conventions

In §3 and onwards we use the following conventions for the type IIB field equations. Since the only non-zero fields are the metric and the four-form potential we need only the Einstein equation which is

$$R_{mn} = \tfrac{1}{96} f_{pqrsm} f^{pqrs}{}_n \tag{A.1}$$

where we must also impose the self-duality constraint on f_{mnpqr}. The five-form f_{mnpqr} is defined in terms of the RR 4-form C_{mnpq} as

$$f_{mnpqr} = 5\partial_{[m}C_{npqr]} \tag{A.2}$$

where here and elsewhere square brackets denote antisymmetrisation with unit weight. Note that this normalisation of the RR field is consistent with that appearing in the D-brane action (2.1). With this truncation of the IIB equations the supersymmetry transformation for the dilatino λ is zero automatically and the gravitino ψ_m variation is

$$\delta\psi_m = (D_m\epsilon + \tfrac{i}{1920}\Gamma^{pqrst}\Gamma_m f_{pqrst}\epsilon). \tag{A.3}$$

We use the following spinor conventions. We work with a mostly positive Lorentz metric η_{mn} and Dirac γ-matrices obeying $\{\gamma_m, \gamma_n\} = 2\eta_{mn}$. The unit normalised matrices $\gamma_{a_1 \ldots a_n}$ are defined by

$$\gamma_{a_1 \ldots a_n} = \gamma_{[a_1} \cdots \gamma_{a_n]}. \tag{A.4}$$

We reserve γ for tangent space Dirac matrices and Γ for curved space matrices.

B Kappa-symmetry and worldvolume flux

We show in this appendix that the kappa symmetry projection (4.19) for the embedding with flux is related to the kappa symmetry projection for the embedding with no flux by a similarity transformation. This is a general property of kappa symmetry projectors and is discussed in [44]. The novelty in our case is that the embedding itself depends explicitly on the flux. This introduces an additional dependence on the flux, through the dependence of the Killing spinors on the spacetime coordinates.

The kappa symmetry projection in the case of no flux is given by

$$\Gamma' = \gamma^{012489}KI \tag{B.1}$$

The similarity transformation that relates (B.1) to the projector with flux is

$$\Gamma = e^{-a/2}\Gamma' e^{a/2} \tag{B.2}$$

where

$$a = \arctan(q)(\gamma^{89}K - \gamma^{34}) \tag{B.3}$$

To prove (B.2) we first work out $e^{a/2}$. This can be done by observing that $\gamma^{89}K$ and γ^{34} square to minus one and commute with each other. After some algebra one obtains,

$$e^{a/2} = \mathcal{P}'_- + \frac{1}{\sqrt{1+q^2}}(1 - q\gamma^{34})\mathcal{P}'_+ \tag{B.4}$$

where

$$\mathcal{P}'_\pm = \tfrac{1}{2}(1 \pm \gamma^{3489} K). \tag{B.5}$$

Using both this result and the fact that Γ' commutes with $\mathcal{P}'_\pm$ but anticommutes with γ^{34} leads (after some more algebra) to (B.2).

Let $\epsilon(x_0, q)$ the Killing spinor evaluated on the embedding surface $x = x_0 - q/u$. Then (B.2) implies that for any solution of the kappa symmetry projection with flux

$$\Gamma\epsilon(x_0, q) = \epsilon(x_0, q), \tag{B.6}$$

there is a solution of

$$\Gamma'\epsilon'(x_0, q) = \epsilon'(x_0, q) \tag{B.7}$$

with $\epsilon'(x_0, q) = e^{a/2}\epsilon(x_0, q)$. We stress that the equation (B.7) is distinct from the equation one would get by considering the zero-flux embeddings. In that case one would have

$$\Gamma'\epsilon(x_0) = \epsilon(x_0) \tag{B.8}$$

where $\epsilon(x_0)$ is the target space Killing vector evaluated on the embedding surface $x = x_0$. Even though it is Γ' that features in both of (B.7) and (B.8), the spinors involved are different.

Clearly, (B.6) and (B.7) are equivalent equations, so one may choose to work with either of them. In (B.7) the kappa symmetry projector is simpler, but the spinor more complicated. In the main text we chose to work with (B.6).

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